Chapter 3

PREFERENCES AND UTILITY

Axioms of Rational Choice (理性选择公理)

- Completeness（完备性）
  - if A and B are any two situations, an individual can always specify exactly one of these possibilities:
    - A is preferred to B
    - B is preferred to A
    - A and B are equally attractive

Axioms of Rational Choice

- Transitivity （传递性）
  - if A is preferred to B, and B is preferred to C, then A is preferred to C
  - assumes that the individual’s choices are internally consistent

Axioms of Rational Choice

- Continuity （连续性）
  - if A is preferred to B, then situations suitably “close to” A must also be preferred to B
  - used to analyze individuals’ responses to relatively small changes in income and prices

Utility（效用）

- Given these assumptions, it is possible to show that people are able to rank in order all possible situations from least desirable to most
- Jeremy Benthan first called this ranking utility
  - if A is preferred to B, then the utility assigned to A exceeds the utility assigned to B

$$U(\mathrm{A}) > U(\mathrm{B})$$

Utility

- Utility rankings are ordinal in nature
  - they record the relative desirability of commodity bundles
- Because utility measures are not unique, it makes no sense to consider how much more utility is gained from A than from B
- It is also impossible to compare utilities between people

Utility

- Utility is affected by the consumption of physical commodities, psychological attitudes, peer group pressures, personal experiences, and the general cultural environment
- Economists generally devote attention to quantifiable options while holding constant the other things that affect utility
  - *ceteris paribus* assumption

Utility

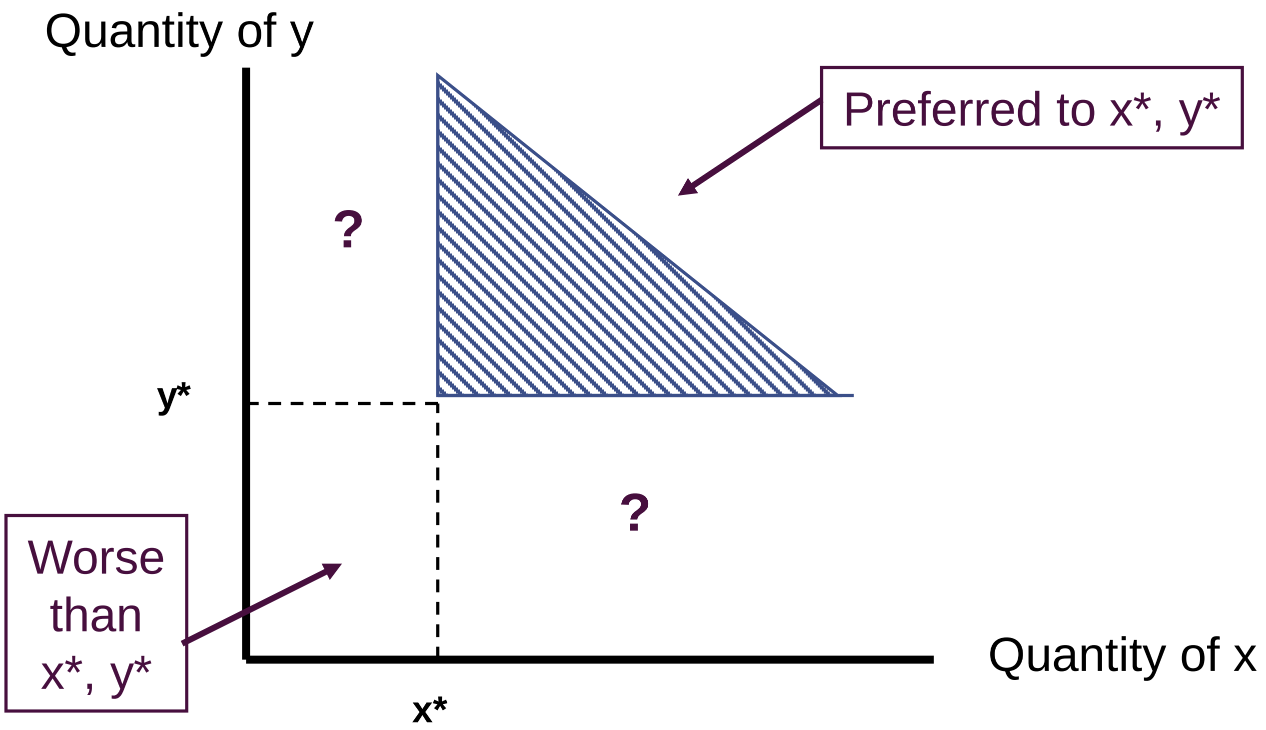
- Assume that an individual must choose among consumption goods $x_1, x_2, \ldots, x_n$
- The individual's rankings can be shown by a utility function (cardinal sense) of the form:

$$\text{utility} = U(x_1, x_2, \ldots, x_n; \text{other things})$$

  - this function is unique up to an order-preserving transformation
  - Sometimes "utility" simply means the value of the utility function. It reflects the "degree of satisfaction."

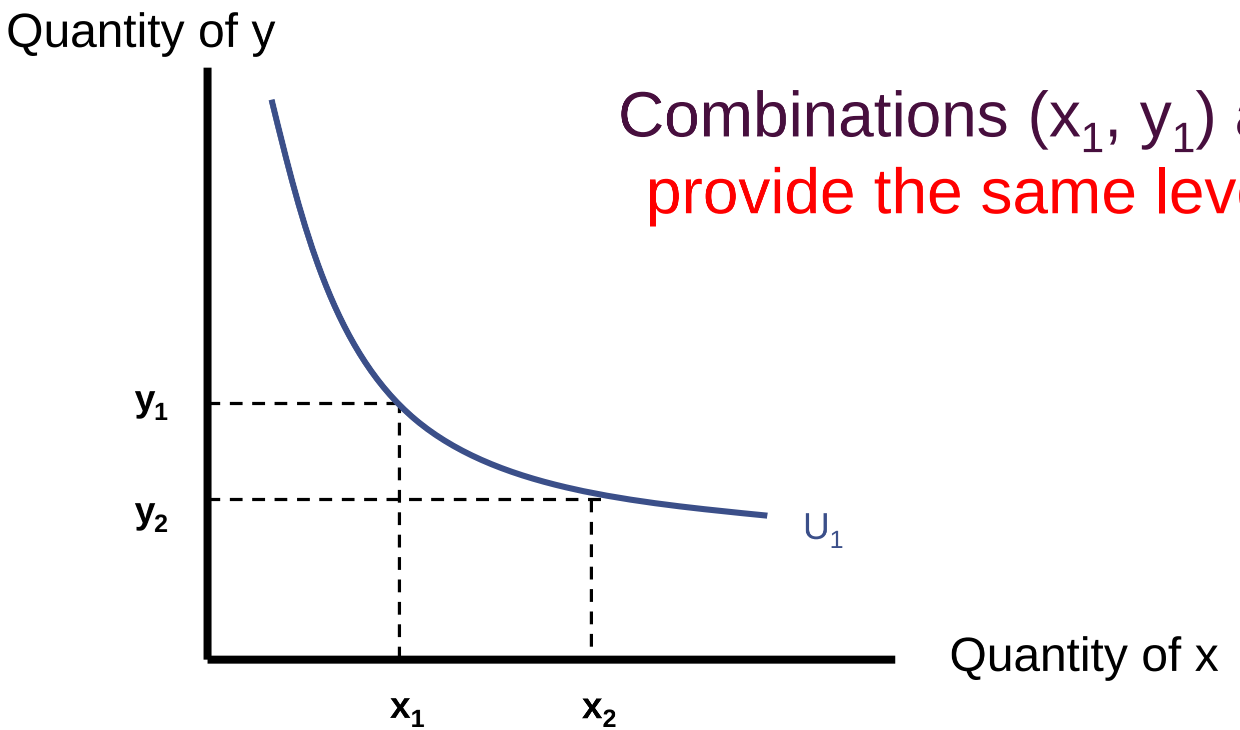
Economic Goods

- In the utility function, the *x*'s are assumed to be "goods"
  – more is preferred to less



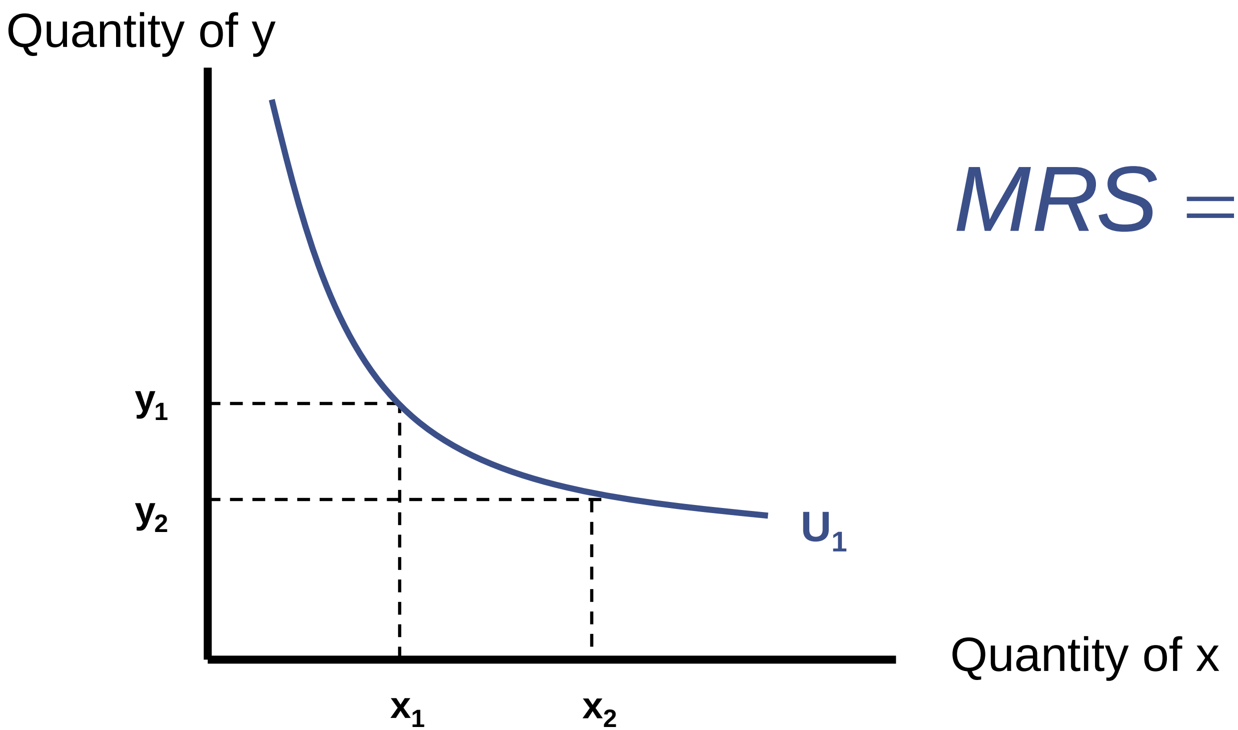
Indifference Curves （无差异曲线）

- An indifference curve shows a set of consumption bundles among which the individual is indifferent



Marginal Rate of Substitution

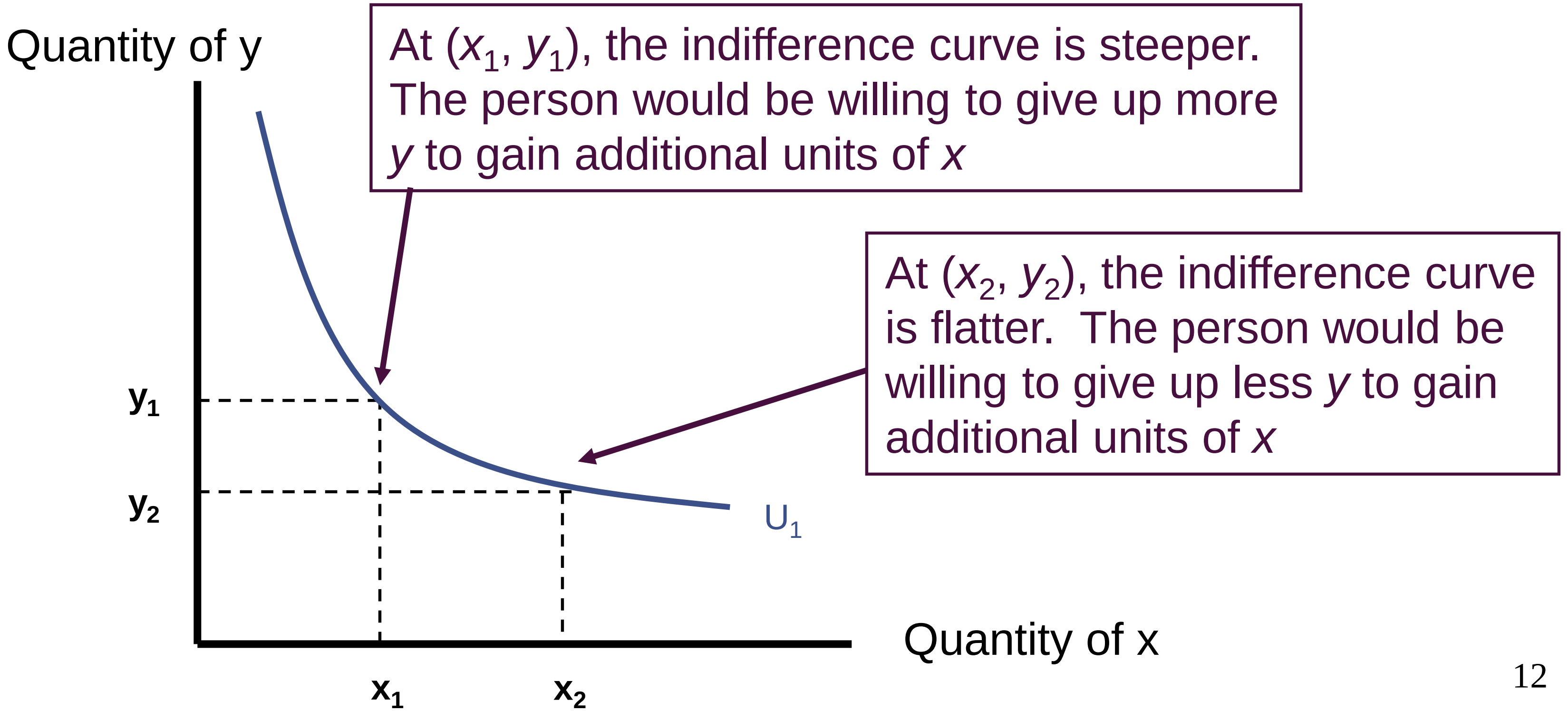
- The negative of the slope of the indifference curve at any point is called the marginal rate of substitution (*MRS* 边际替代率）



$$MRS = -\left.\frac{dy}{dx}\right|_{U=U_1}$$

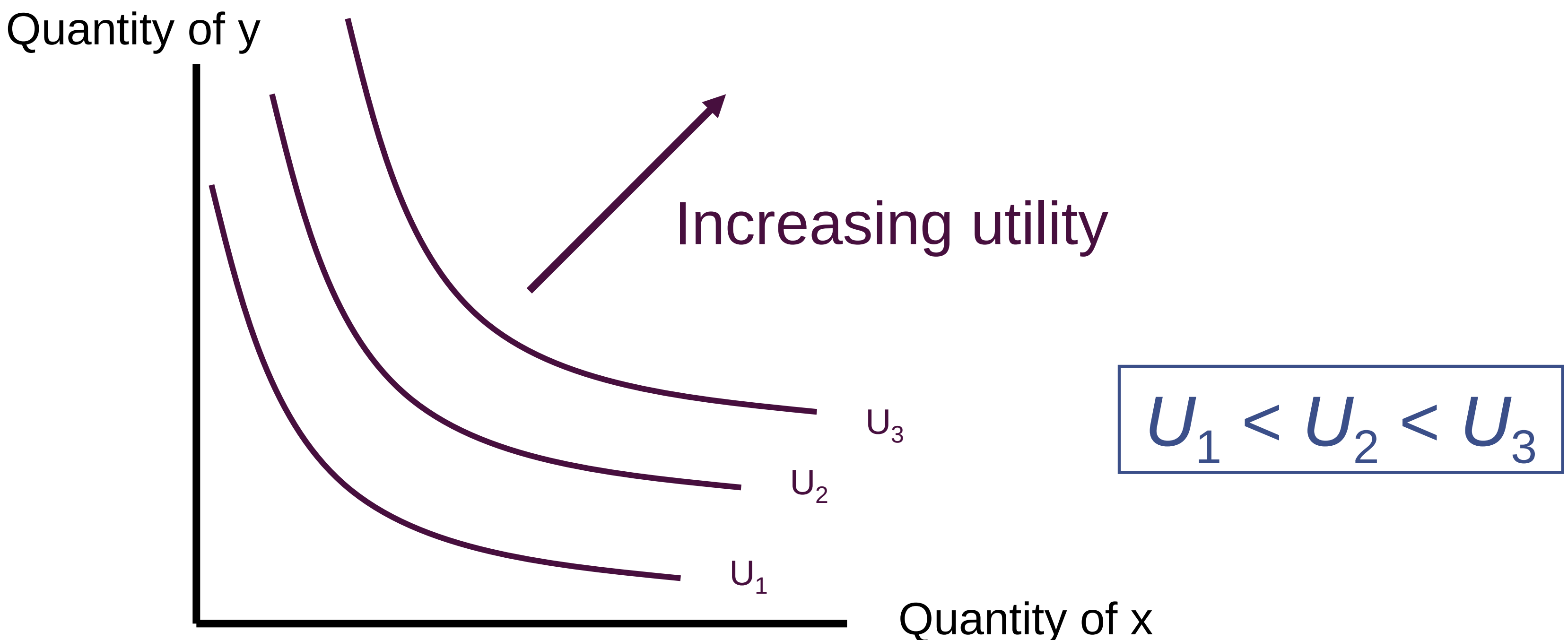
Marginal Rate of Substitution

- MRS changes as *x* and *y* change
  - reflects the individual's willingness to trade *y* for *x*



Indifference Curve Map

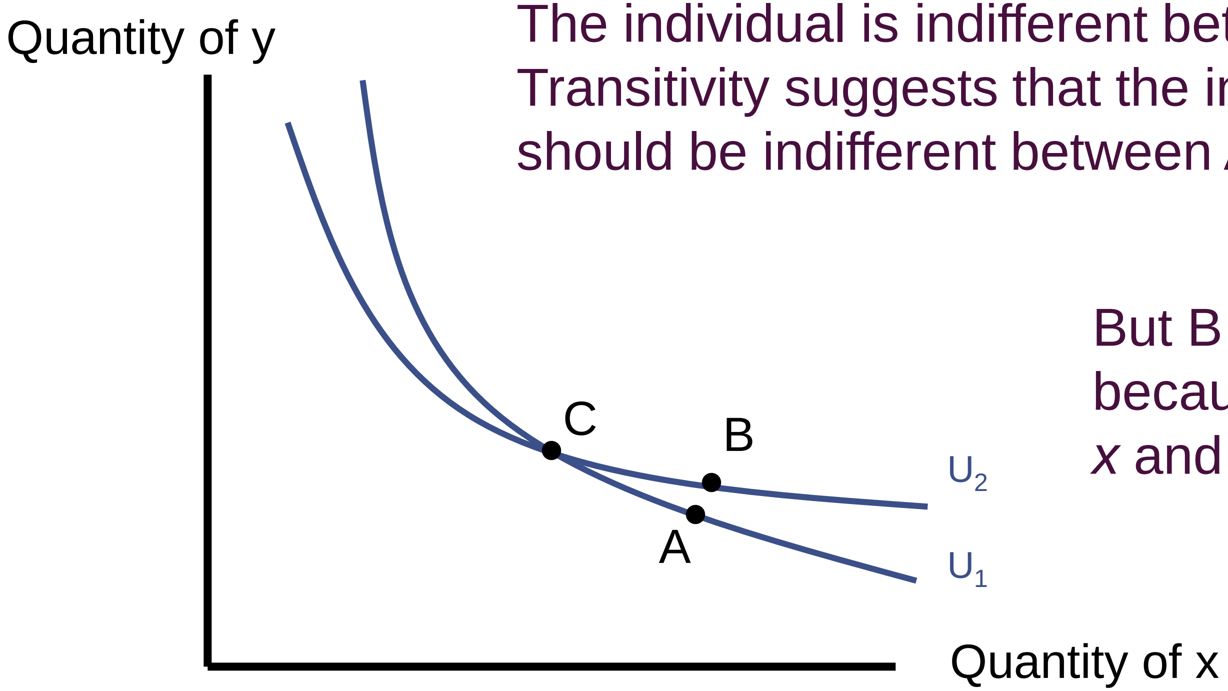
- Each point must have an indifference curve through it



Transitivity

- Can any two of an individual's indifference curves intersect?

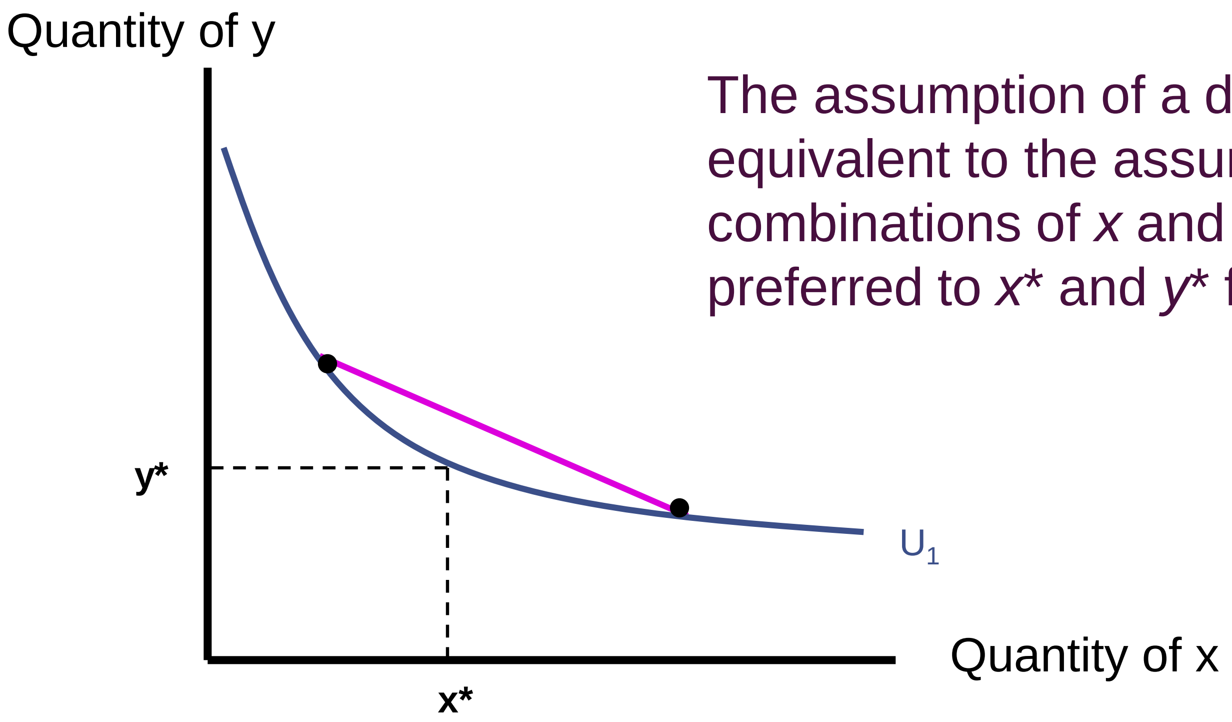
The individual is indifferent between A and C.
The individual is indifferent between B and C.
Transitivity suggests that the individual should be indifferent between A and B



But B is preferred to A because B contains more *x* and *y* than A

Convexity

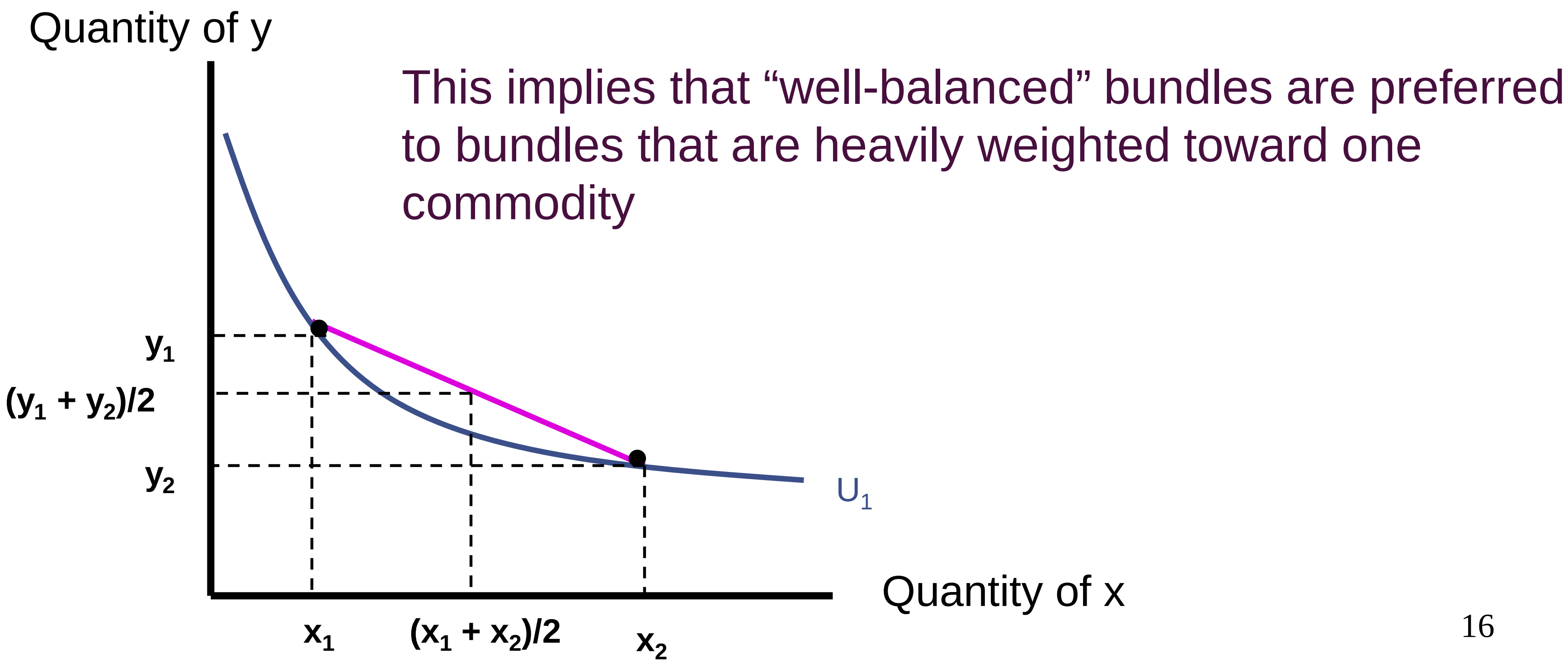
- A set of points is convex if any two points can be joined by a straight line that is contained completely within the set



The assumption of a diminishing MRS is equivalent to the assumption that all combinations of *x* and *y* which are preferred to *x** and *y** form a convex set

Convexity

- If the indifference curve is convex, then the combination $(x_1 + x_2)/2$, $(y_1 + y_2)/2$ will be preferred to either (x_1,y_1) or (x_2,y_2)



Utility and the MRS

- Suppose an individual's preferences for hamburgers (*y*) and soft drinks (*x*) can be represented by

$$\text{utility} = 10 = \sqrt{x \cdot y}$$

- Solving for *y*, we get

$$y = 100/x$$

- Solving for MRS = -*dy*/*dx*:

$$MRS = -dy/dx = 100/x^2$$

Utility and the MRS

$$MRS = -dy/dx = 100/x^2$$

- Note that as x rises, *MRS* falls
  - when $x = 5$, *MRS* = 4
  - when $x = 20$, *MRS* = 0.25

Marginal Utility

- Suppose that an individual has a utility function of the form

$$\text{utility} = U(x,y)$$

- The total differential of *U* is

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy$$

- Along any indifference curve, utility is constant ($dU = 0$)

Deriving the *MRS*

- Therefore, we get:

$$MRS = -\left.\frac{dy}{dx}\right|_{U=\text{constant}} = \frac{\dfrac{\partial U}{\partial x}}{\dfrac{\partial U}{\partial y}}$$

- *MRS* is the ratio of the marginal utility of *x* to the marginal utility of *y*

Diminishing Marginal Utility and the *MRS*

- Intuitively, it seems that the assumption of decreasing marginal utility is related to the concept of a diminishing *MRS*
  - diminishing *MRS* requires that the utility function be **quasi-concave**
    - this is independent of how utility is measured
  - diminishing marginal utility depends on how utility is measured
- Thus, these two concepts are different: cardinal vs ordinal.

Convexity of Indifference Curves

- Suppose that the utility function is

$$\text{utility} = \sqrt{x \cdot y}$$

- We can simplify the algebra by taking the logarithm of this function

$$U^*(x,y) = \ln[U(x,y)] = 0.5 \ln x + 0.5 \ln y$$

Convexity of Indifference Curves

- Thus,

$$MRS = \frac{\frac{\partial U^*}{\partial x}}{\frac{\partial U^*}{\partial y}} = \frac{\frac{0.5}{x}}{\frac{0.5}{y}} = \frac{y}{x}$$

Convexity of Indifference Curves

- If the utility function is

$$U(x,y) = x + xy + y$$

- There is no advantage to transforming this utility function, so

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{1+y}{1+x}$$

Convexity of Indifference Curves

- Suppose that the utility function is

$$\text{utility} = \sqrt{x^2 \cdot y^2}$$

- For this example, it is easier to use the transformation

$$U^*(x,y) = [U(x,y)]^2 = x^2 + y^2$$

Convexity of Indifference Curves

- Thus,

$$MRS = \frac{\frac{\partial U^*}{\partial x}}{\frac{\partial U^*}{\partial y}} = \frac{2x}{2y} = \frac{x}{y}$$

Examples of Utility Functions

- Cobb-Douglas Utility

$$\text{utility} = U(x,y) = x^{\alpha}y^{\beta}$$

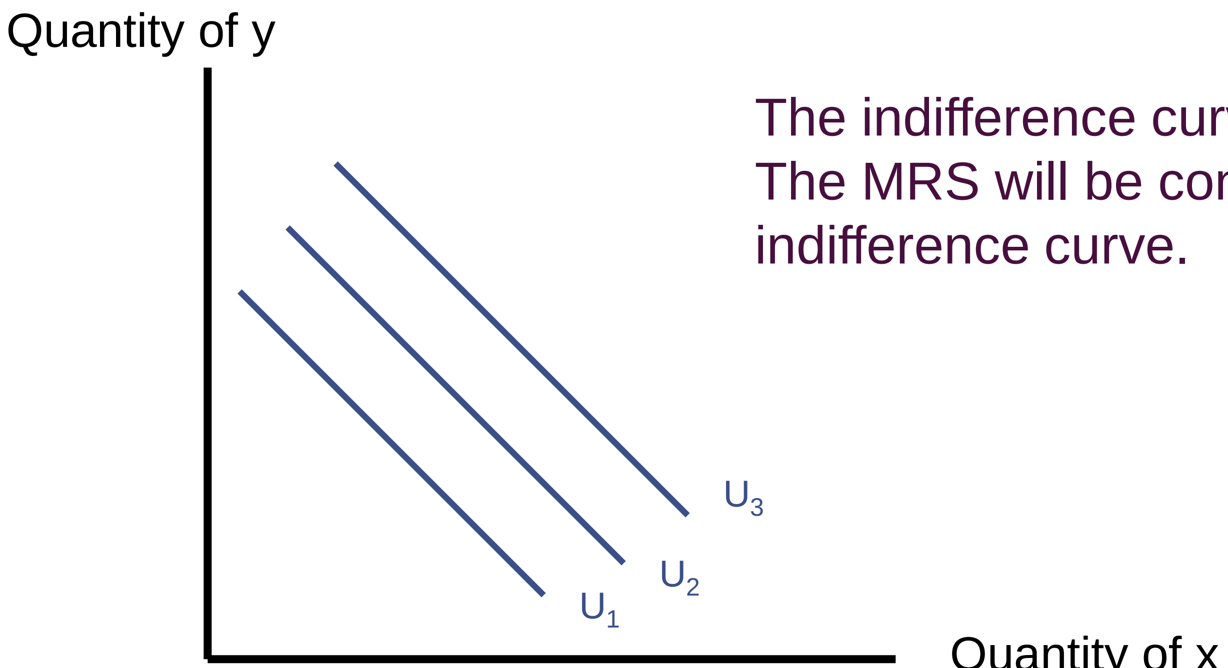
where α and β are positive constants

– The relative sizes of α and β indicate the relative importance of the goods

Examples of Utility Functions

- Perfect Substitutes

$$\text{utility} = U(x,y) = \alpha x + \beta y$$

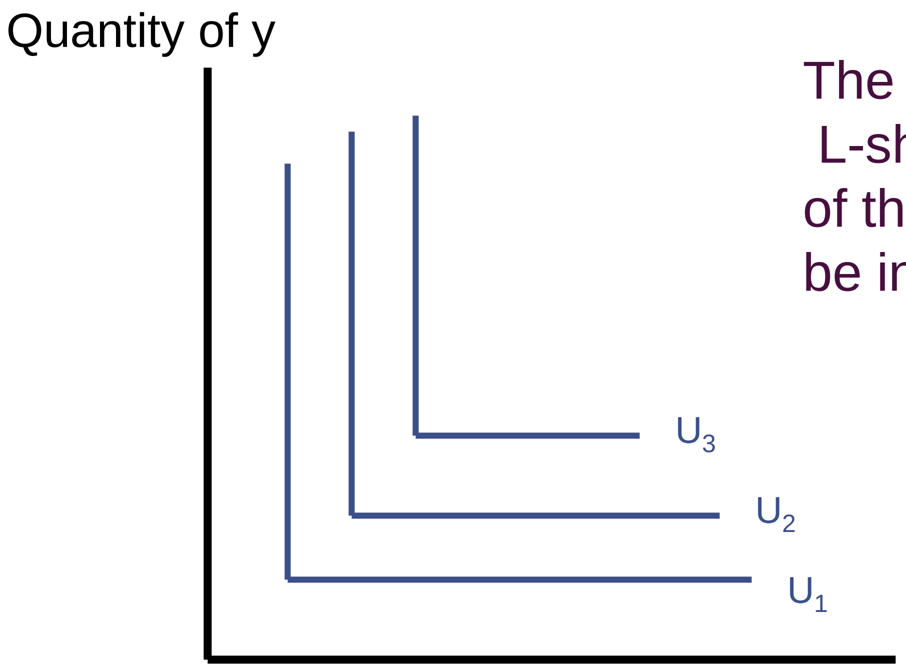


The indifference curves will be linear. The MRS will be constant along the indifference curve.

Examples of Utility Functions

- Perfect Complements

$$\text{utility} = U(x,y) = \min(\alpha x, \beta y)$$



The indifference curves will be L-shaped. Only by choosing more of the two goods together can utility be increased.

Examples of Utility Functions

- CES Utility (Constant elasticity of substitution)

$$\text{utility} = U(x,y) = x^\delta/\delta + y^\delta/\delta$$

when $\delta \neq 0$ and

$$\text{utility} = U(x,y) = \ln x + \ln y$$

when $\delta = 0$

  - Perfect substitutes $\Rightarrow \delta = 1$
  - Cobb-Douglas $\Rightarrow \delta = 0$
  - Perfect complements $\Rightarrow \delta = -\infty$

Examples of Utility Functions

- CES Utility (Constant elasticity of substitution)
  - **The elasticity of substitution (σ) is equal to 1/(1 - δ)**
    - Perfect substitutes ⇒ $\sigma = \infty$
    - Fixed proportions ⇒ $\sigma = 0$

Homothetic Preferences

- If the *MRS* depends only on the ratio of the amounts of the two goods, not on the quantities of the goods, the utility function is homothetic
  - Perfect substitutes $\Rightarrow$ *MRS* is the same at every point
  - Perfect complements $\Rightarrow$ $MRS = \infty$ if $y/x > \alpha/\beta$, undefined if $y/x = \alpha/\beta$, and $MRS = 0$ if $y/x < \alpha/\beta$

Homothetic Preferences

- For the general Cobb-Douglas function, the *MRS* can be found as

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{\alpha x^{\alpha-1} y^{\beta}}{\beta x^{\alpha} y^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$

Nonhomothetic Preferences

- Some utility functions do not exhibit homothetic preferences

$$\text{utility} = U(x,y) = x + \ln y$$

$$MRS = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = \frac{1}{\frac{1}{y}} = y$$

The Many-Good Case

- Suppose utility is a function of *n* goods given by

$$\text{utility} = U(x_1, x_2, \ldots, x_n)$$

- The total differential of *U* is

$$dU = \frac{\partial U}{\partial x_1}dx_1 + \frac{\partial U}{\partial x_2}dx_2 + \ldots + \frac{\partial U}{\partial x_n}dx_n$$

The Many-Good Case

- We can find the *MRS* between any two goods by setting $dU = 0$

$$dU = 0 = \frac{\partial U}{\partial x_i} dx_i + \frac{\partial U}{\partial x_j} dx_j$$

- Rearranging, we get

$$MRS(x_i \text{ for } x_j) = -\frac{dx_j}{dx_i} = \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_j}}$$

Multigood Indifference Surfaces

- We will define an indifference surface as being the set of points in *n* dimensions that satisfy the equation

$$U(x_1, x_2, \ldots x_n) = k$$

where *k* is any preassigned constant

Multigood Indifference Surfaces

- If the utility function is quasi-concave, the set of points for which $U \geq k$ will be convex
  - all of the points on a line joining any two points on the $U = k$ indifference surface will also have $U \geq k$

Important Points to Note:

- If individuals obey certain behavioral postulates (公设), they will be able to rank all commodity bundles
  - the ranking can be represented by a utility function
  - in making choices, individuals will act as if they were maximizing this function
- Utility functions for two goods can be illustrated by an indifference curve map

Important Points to Note:

- The negative of the slope of the indifference curve measures the marginal rate of substitution (*MRS*)
  - the rate at which an individual would trade an amount of one good (*y*) for one more unit of another good (*x*)
- *MRS* decreases as *x* is substituted for *y*
  - individuals prefer some balance in their consumption choices

Important Points to Note:

- A few simple functional forms can capture important differences in individuals' preferences for two (or more) goods
  - Cobb-Douglas function
  - linear function (perfect substitutes)
  - fixed proportions function (perfect complements)
  - CES function
    - includes the other three as special cases

Important Points to Note:

- It is a simple matter to generalize from two-good examples to many goods
  - studying peoples' choices among many goods can yield many insights
  - the mathematics of many goods is not especially intuitive, so we will rely on two-good cases to build intuition