Chapter 5

INCOME AND SUBSTITUTION EFFECTS

Demand Functions

- The optimal levels of $x_1,x_2,\ldots,x_n$ can be expressed as functions of all prices and income
- These can be expressed as *n* demand functions of the form:

$$x_1^* = d_1(p_1,p_2,\ldots,p_n,I)$$
$$x_2^* = d_2(p_1,p_2,\ldots,p_n,I)$$
$$\vdots$$
$$x_n^* = d_n(p_1,p_2,\ldots,p_n,I)$$

Demand Functions

- If there are only two goods (*x* and *y*), we can simplify the notation

$$x^* = x(p_x, p_y, I)$$

$$y^* = y(p_x, p_y, I)$$

- Prices and income are exogenous
  - the individual has no control over these parameters

Homogeneity

- If we were to double all prices and income, the optimal quantities demanded will not change
  - the budget constraint is unchanged

$$x_i^* = d_i(p_1,p_2,\ldots,p_n,I) = d_i(tp_1,tp_2,\ldots,tp_n,tI)$$

- Individual demand functions are homogeneous of degree zero in all prices and income

Homogeneity

- With a Cobb-Douglas utility function

$$\text{utility} = U(x,y) = x^{0.3}y^{0.7}$$

the demand functions are

$$x^* = \frac{0.3I}{p_x} \qquad\qquad y^* = \frac{0.7I}{p_y}$$

- Note that a doubling of both prices and income would leave x^* and y^* unaffected

Homogeneity

- With a CES utility function

$$\text{utility} = U(x,y) = x^{0.5} + y^{0.5}$$

the demand functions are

$$x^* = \frac{1}{1 + p_x / p_y} \cdot \frac{I}{p_x} \qquad y^* = \frac{1}{1 + p_y / p_x} \cdot \frac{I}{p_y}$$

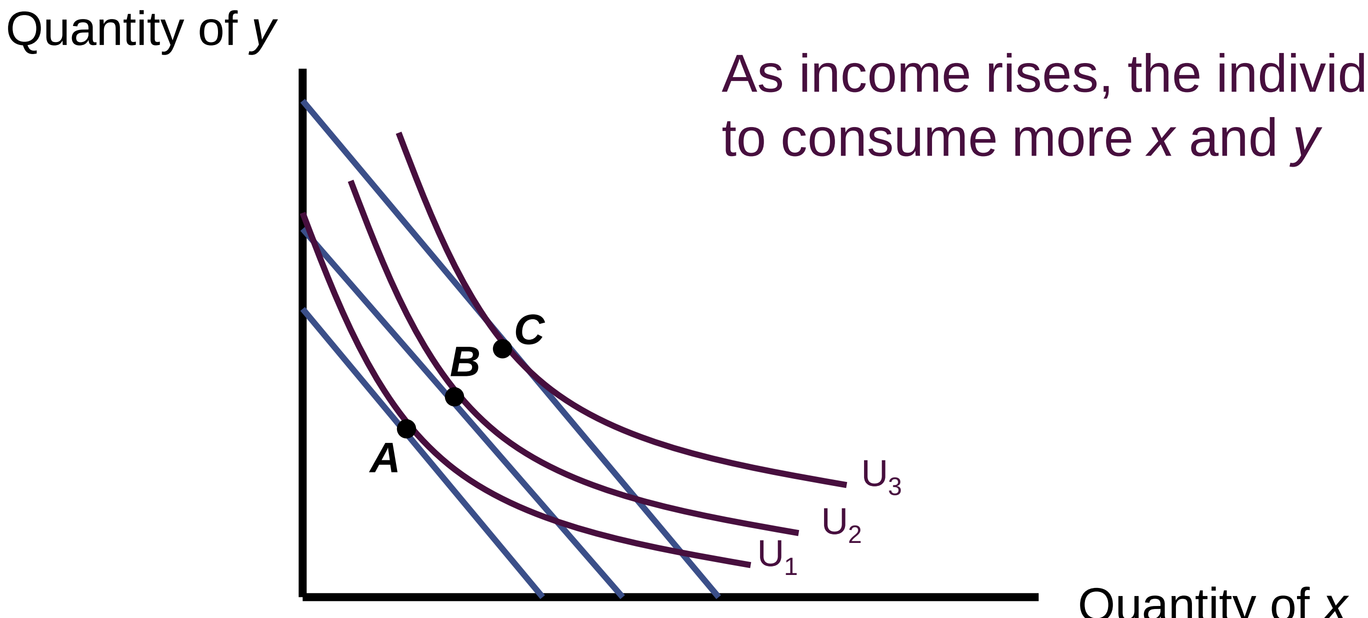
- Note that a doubling of both prices and income would leave x^* and y^* unaffected

Changes in Income

- An increase in income will cause the budget constraint out in a parallel fashion
- Since p_x/p_y does not change, the *MRS* will stay constant as the worker moves to higher levels of satisfaction

Increase in Income

- If both *x* and *y* increase as income rises, *x* and *y* are normal goods (正常品)

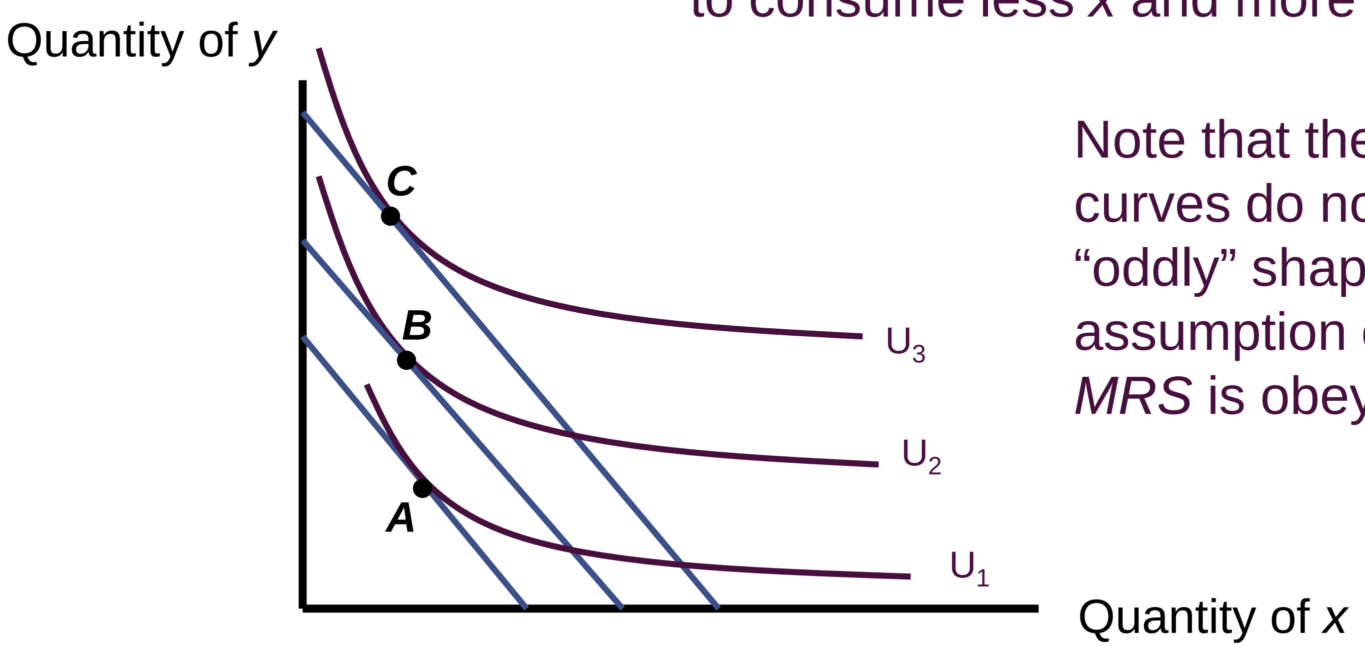


As income rises, the individual chooses to consume more *x* and *y*

Increase in Income

- If *x* decreases as income rises, *x* is an <u>inferior</u> good（劣质品）

As income rises, the individual chooses to consume less *x* and more *y*



Note that the indifference curves do not have to be "oddly" shaped. The assumption of a diminishing *MRS* is obeyed.

Normal and Inferior Goods

- A good x_i for which $\partial x_i/\partial I \geq 0$ over some range of income is a normal good in that range

- A good x_i for which $\partial x_i/\partial I < 0$ over some range of income is an inferior good in that range

Changes in a Good's Price

- A change in the price of a good alters the slope of the budget constraint
  – it also changes the *MRS* at the consumer's utility-maximizing choices
- When the price changes, two effects come into play
  – substitution effect（替代效应）
  – income effect（收入效应）

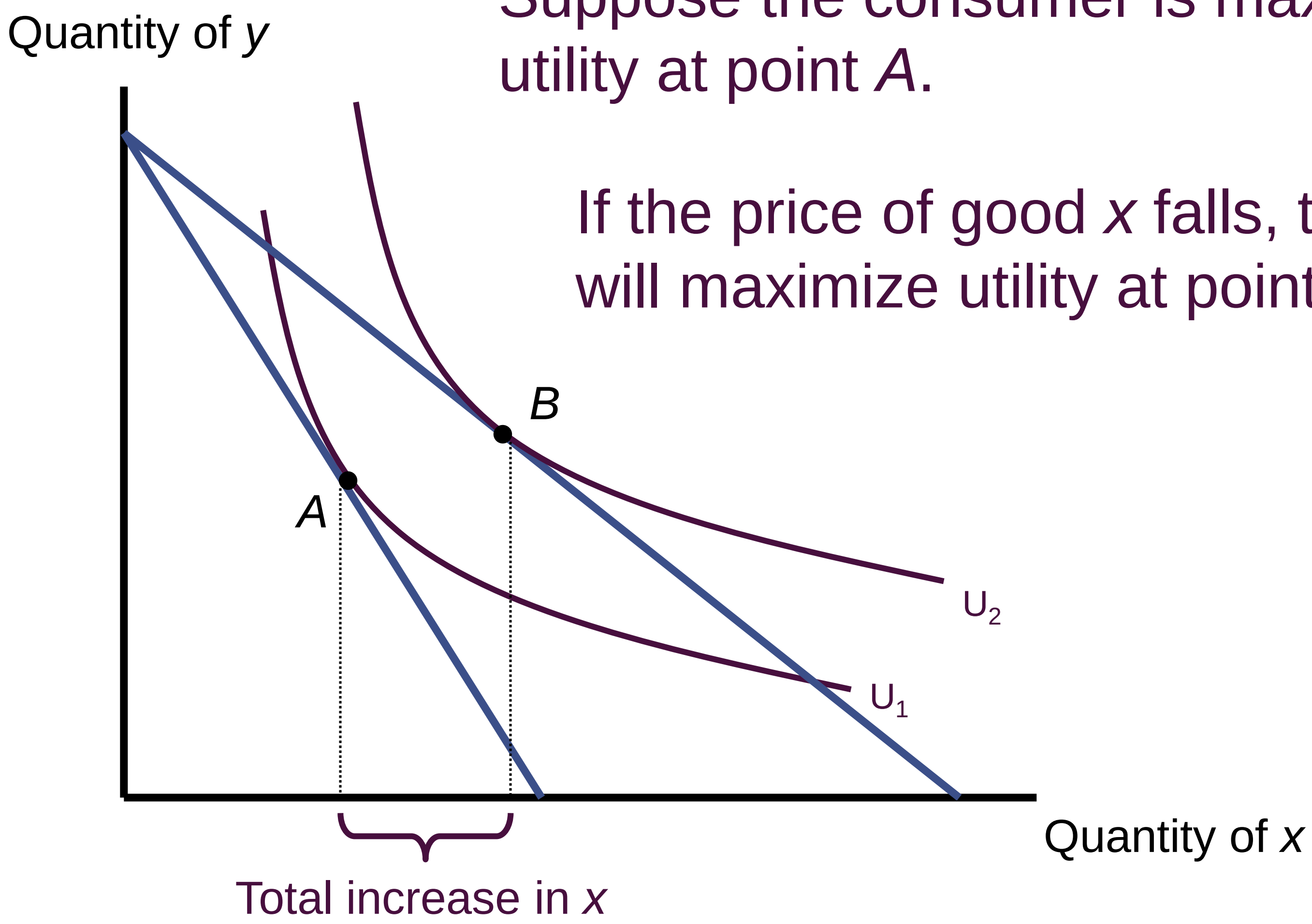
Changes in a Good's Price

- Even if the individual remained on the same indifference curve when the price changes, his optimal choice will change because the *MRS* must equal the new price ratio
  - the substitution effect
- The price change alters the individual's "real" income and therefore he must move to a new indifference curve
  - the income effect

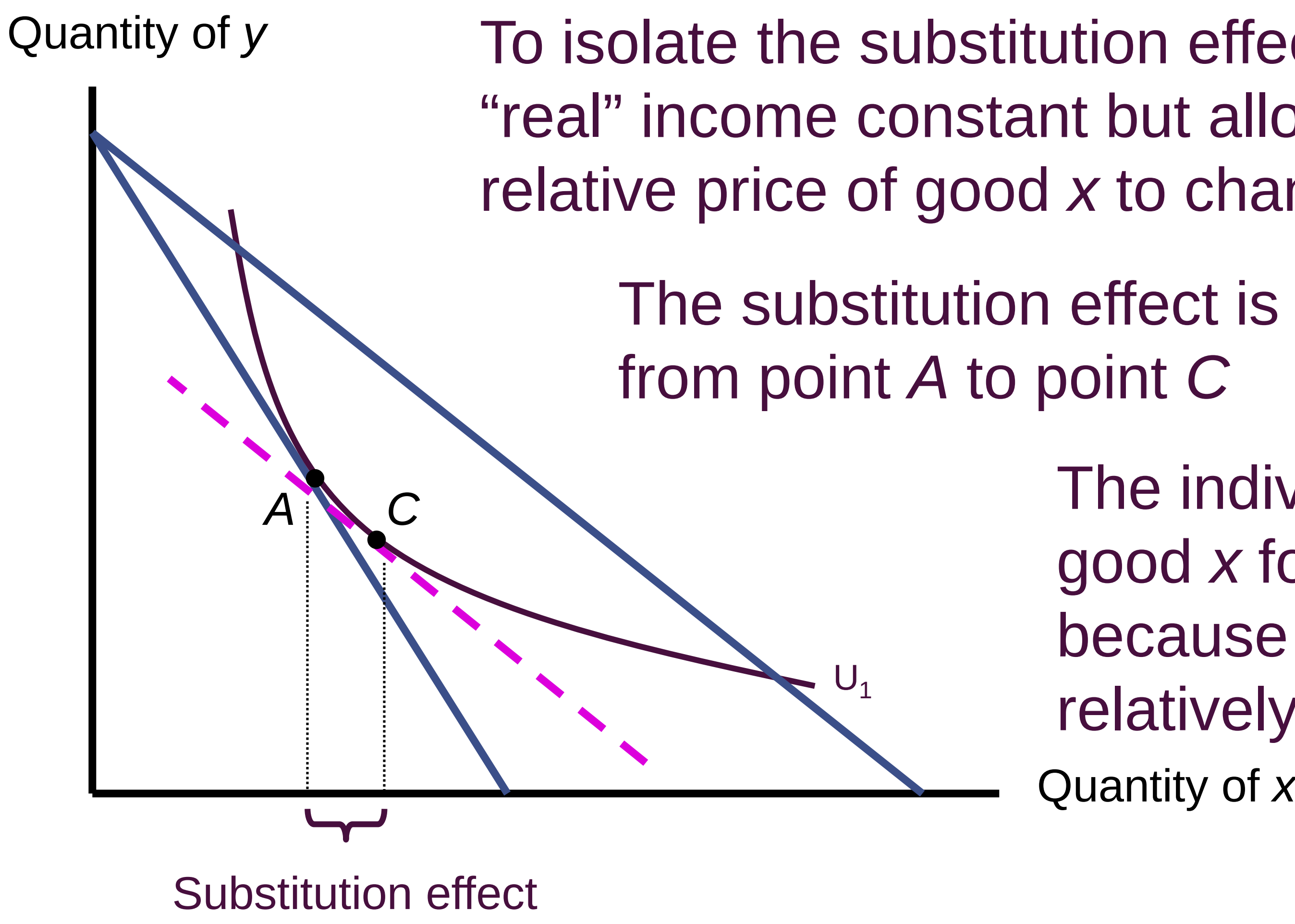
Changes in a Good's Price

Suppose the consumer is maximizing utility at point *A*.

If the price of good *x* falls, the consumer will maximize utility at point *B*.



Changes in a Good's Price

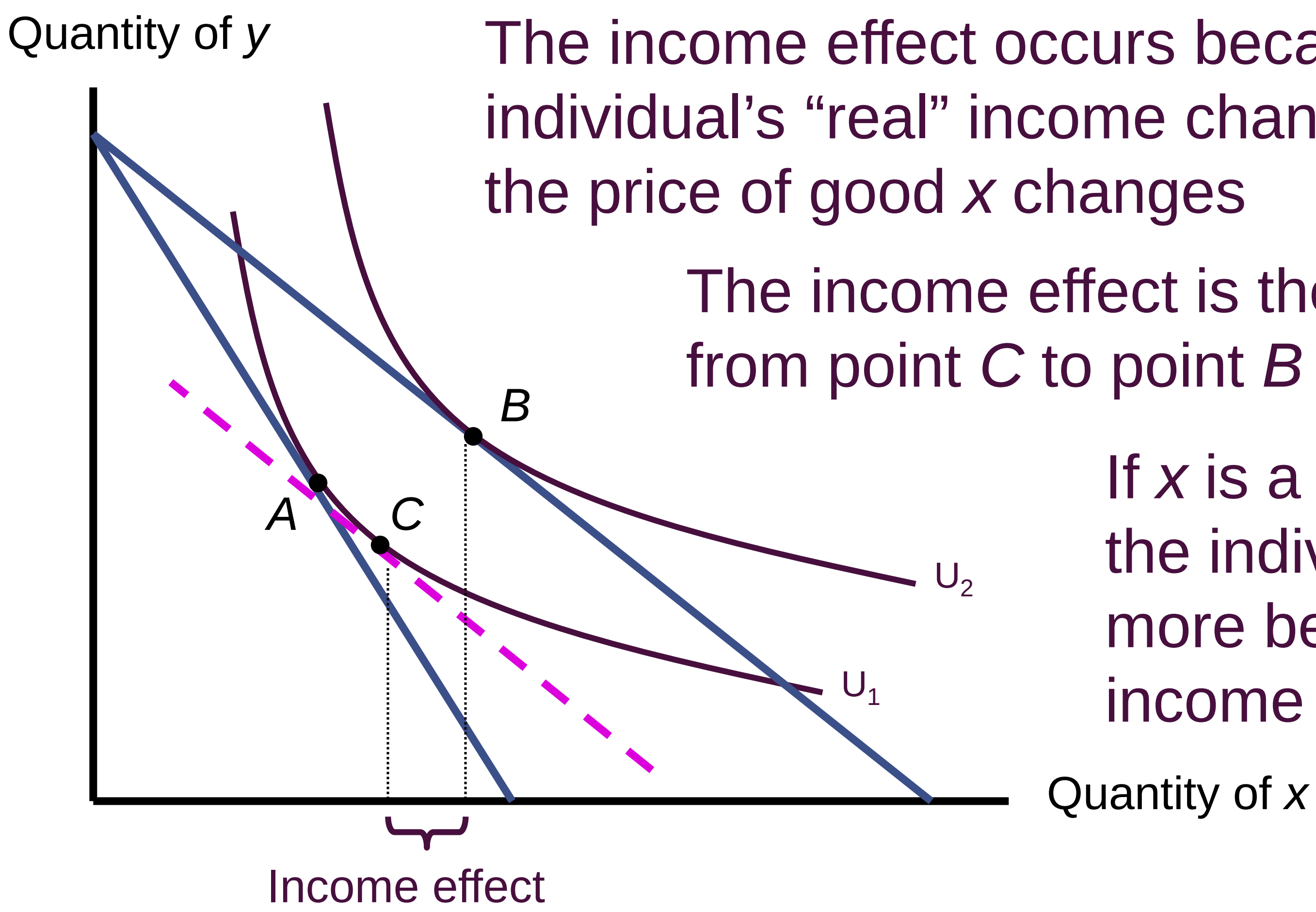


To isolate the substitution effect, we hold "real" income constant but allow the relative price of good *x* to change

The substitution effect is the movement from point *A* to point *C*

The individual substitutes good *x* for good *y* because it is now relatively cheaper

Changes in a Good's Price

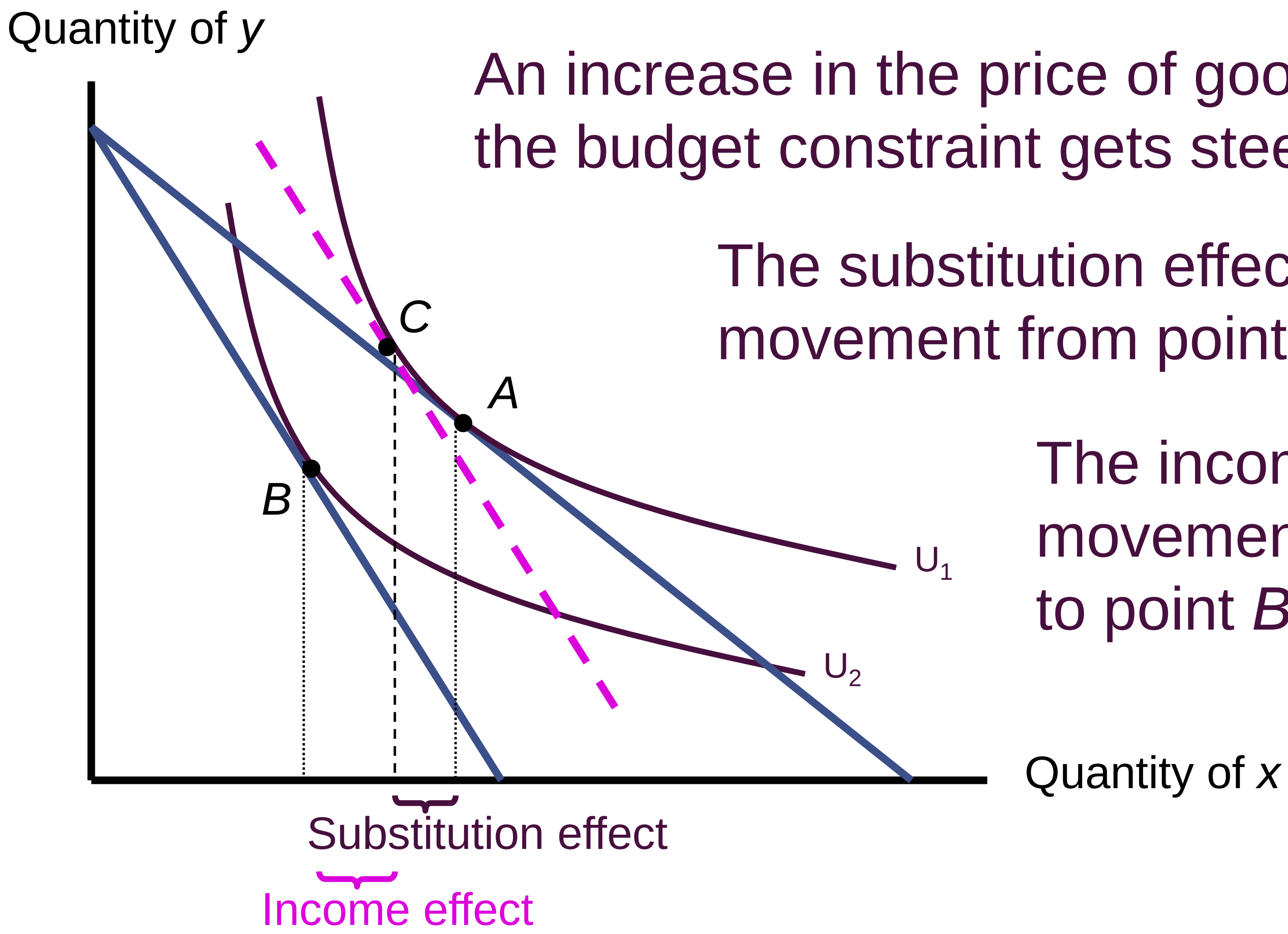


The income effect occurs because the individual's "real" income changes when the price of good *x* changes

The income effect is the movement from point *C* to point *B*

If *x* is a normal good, the individual will buy more because "real" income increased

Changes in a Good's Price



An increase in the price of good *x* means that the budget constraint gets steeper

The substitution effect is the movement from point *A* to point *C*

The income effect is the movement from point *C* to point *B*

Price Changes for Normal Goods

- If a good is normal, substitution and income effects reinforce one another
  - when price falls, both effects lead to a rise in quantity demanded
  - when price rises, both effects lead to a drop in quantity demanded

Price Changes for Inferior Goods

- If a good is inferior, substitution and income effects move in opposite directions
- The combined effect is indeterminate
  - when price rises, the substitution effect leads to a drop in quantity demanded, but the income effect is opposite
  - when price falls, the substitution effect leads to a rise in quantity demanded, but the income effect is opposite

Giffen's Paradox

- If the income effect of a price change is strong enough, there could be a positive relationship between price and quantity demanded
  - an increase in price leads to a drop in real income
  - since the good is inferior, a drop in income causes quantity demanded to rise

A Summary

- Utility maximization implies that (for normal goods) a fall in price leads to an increase in quantity demanded
  - the substitution effect causes more to be purchased as the individual moves along an indifference curve
  - the income effect causes more to be purchased because the resulting rise in purchasing power allows the individual to move to a higher indifference curve

A Summary

- Utility maximization implies that (for normal goods) a rise in price leads to a decline in quantity demanded
  - the substitution effect causes less to be purchased as the individual moves along an indifference curve
  - the income effect causes less to be purchased because the resulting drop in purchasing power moves the individual to a lower indifference curve

A Summary

- Utility maximization implies that (for inferior goods) no definite prediction can be made for changes in price
  - the substitution effect and income effect move in opposite directions
  - if the income effect outweighs the substitution effect, we have a case of Giffen’s paradox

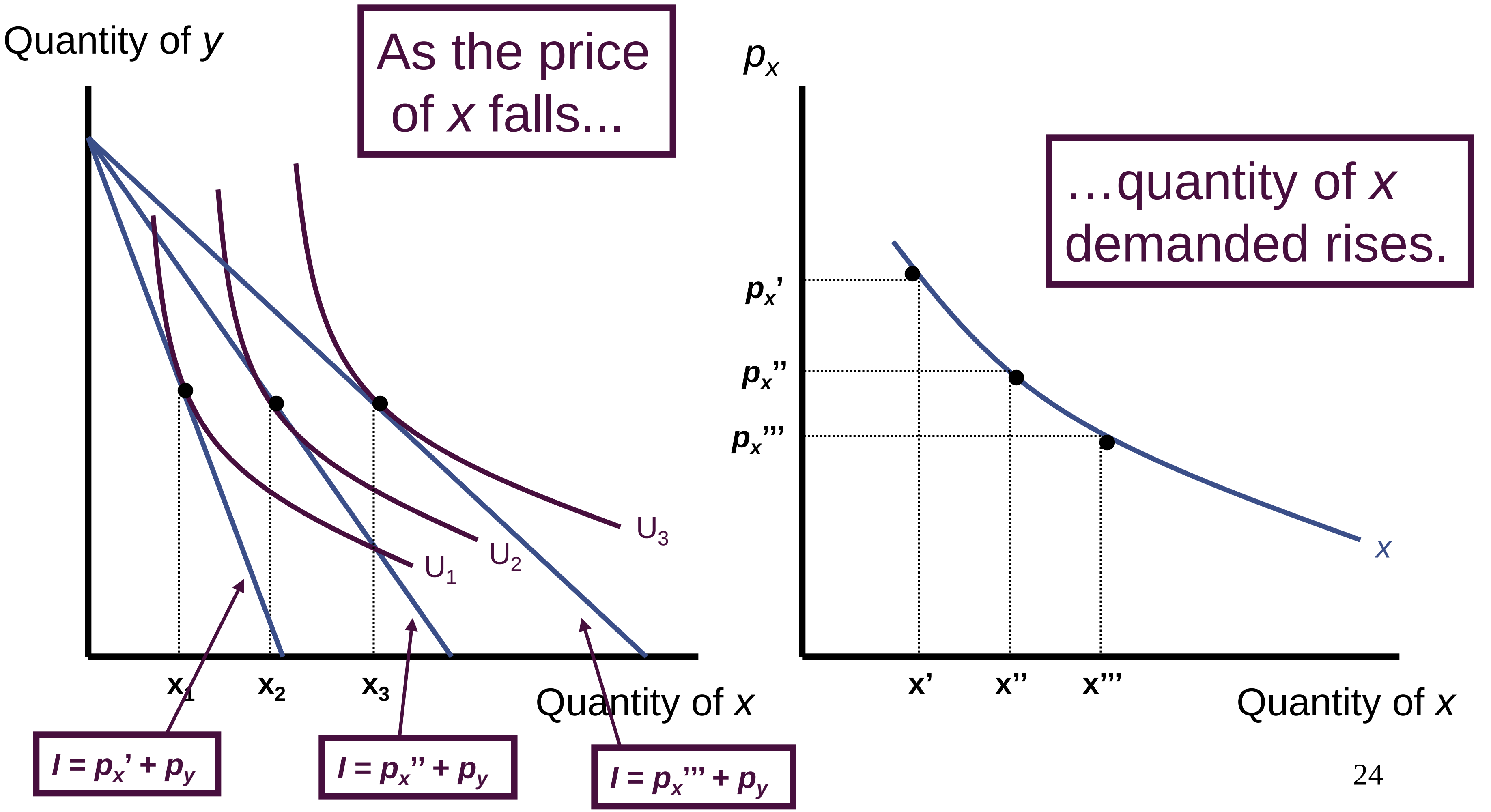
The Individual's Demand Curve

- An individual's demand for *x* depends on preferences, all prices, and income:

$$x^* = x(p_x, p_y, I)$$

- It may be convenient to graph the individual's demand for *x* assuming that income and the price of *y* (p_y) are held constant

The Individual's Demand Curve



The Individual's Demand Curve

- An individual demand curve shows the relationship between the price of a good and the quantity of that good purchased by an individual assuming that all other determinants of demand are held constant

Shifts in the Demand Curve

- Three factors are held constant when a demand curve is derived
  - income
  - prices of other goods (p_y)
  - the individual's preferences
- If any of these factors change, the demand curve will shift to a new position

Shifts in the Demand Curve

- A movement along a given demand curve is caused by a change in the price of the good
  - a change in quantity demanded
- A shift in the demand curve is caused by changes in income, prices of other goods, or preferences
  - a change in demand

Demand Functions and Curves

- We discovered earlier that

$$x^* = \frac{0.3I}{p_x} \qquad y^* = \frac{0.7I}{p_y}$$

- If the individual's income is $100, these functions become

$$x^* = \frac{30}{p_x} \qquad y^* = \frac{70}{p_y}$$

Demand Functions and Curves

- Any change in income will shift these demand curves

Compensated Demand Curves
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- The actual level of utility varies along the demand curve
- As the price of *x* falls, the individual moves to higher indifference curves
  - it is assumed that nominal income is held constant as the demand curve is derived
  - this means that "real" income rises as the price of *x* falls

Compensated Demand Curves

- An alternative approach holds real income (or utility) constant while examining reactions to changes in p_x
  - the effects of the price change are "compensated" so as to constrain the individual to remain on the same indifference curve
  - reactions to price changes include only substitution effects

Compensated Demand Curves

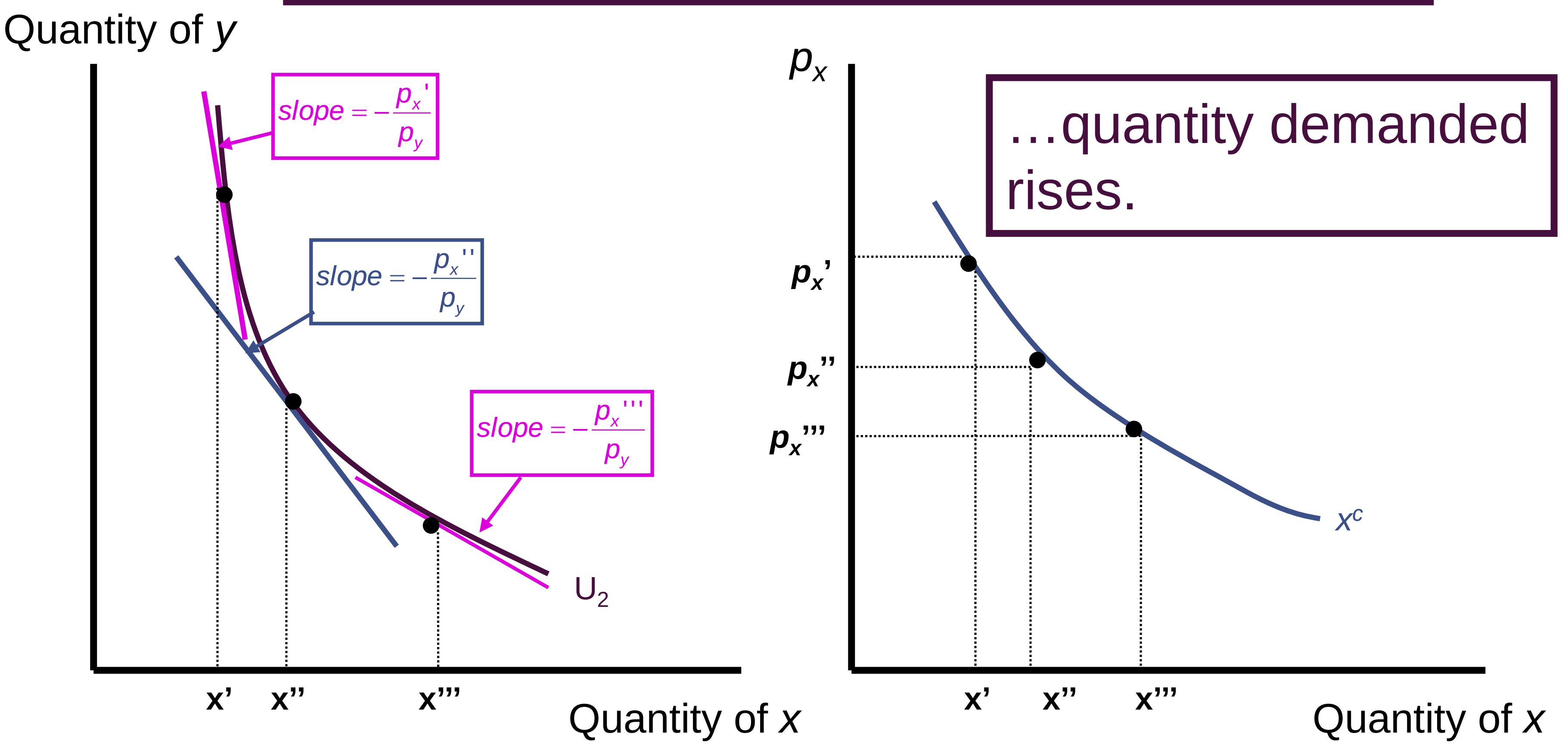
- A compensated (Hicksian) demand curve shows the relationship between the price of a good and the quantity purchased assuming that other prices and utility are held constant
- The compensated demand curve is a two-dimensional representation of the compensated demand function

$$x^* = x^c(p_x, p_y, U)$$

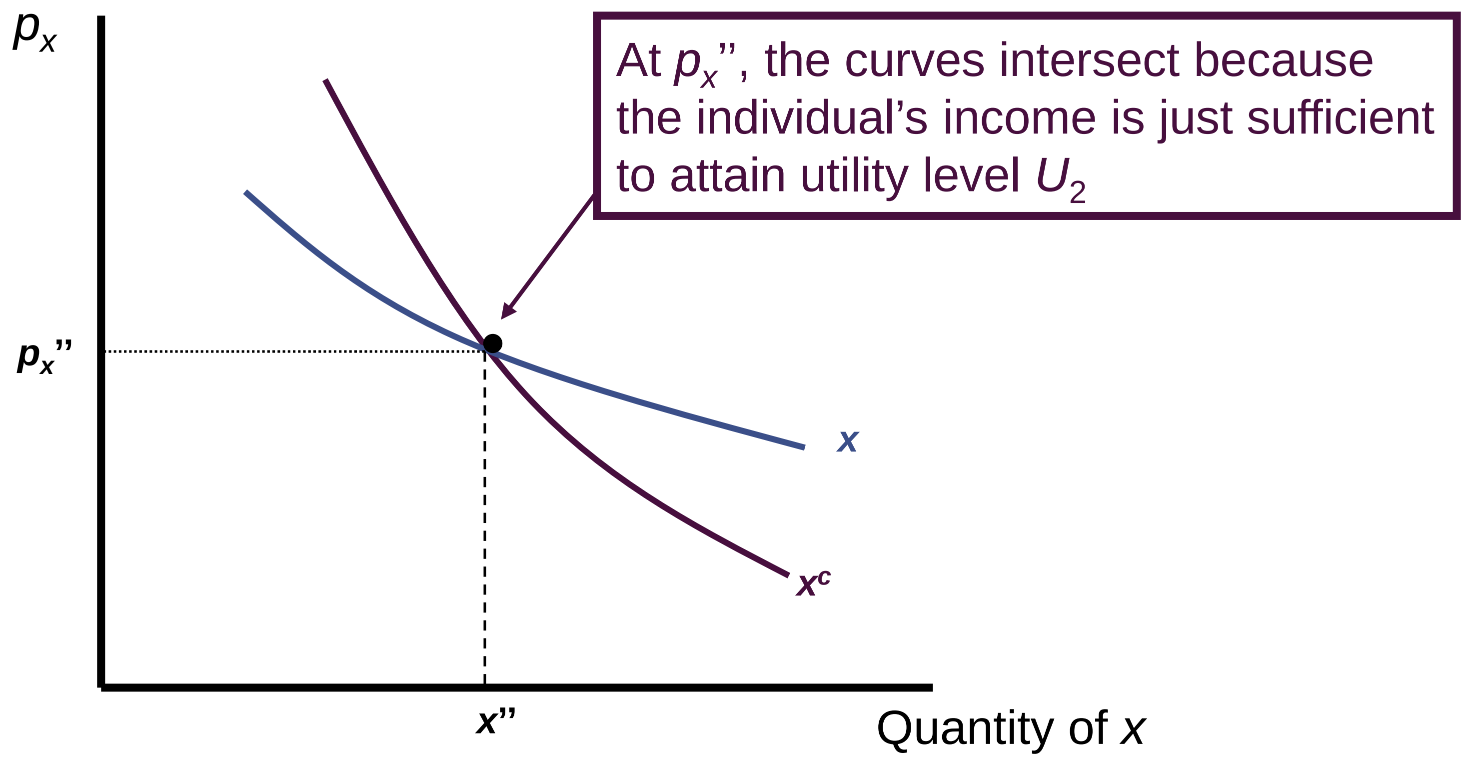
Compensated Demand Curves

Holding utility constant, as price falls...

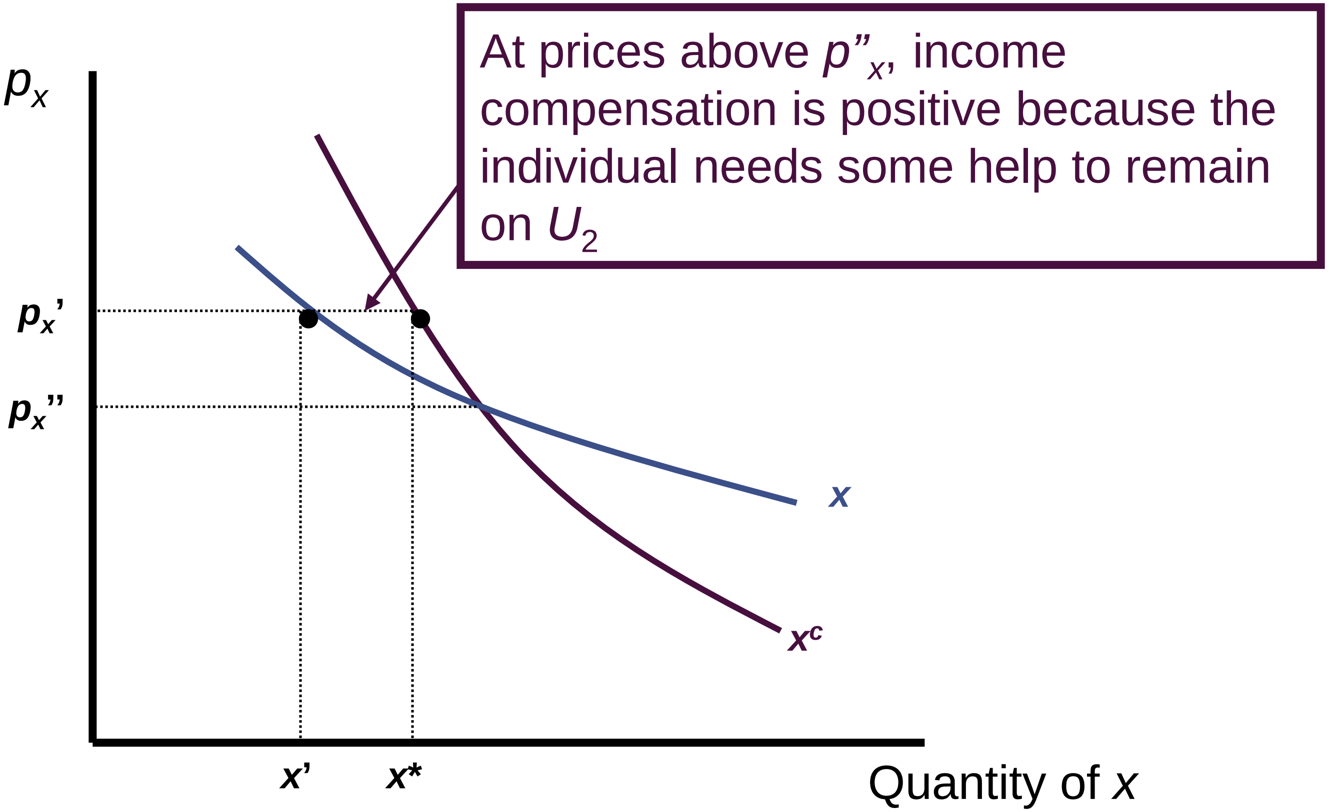
…quantity demanded rises.



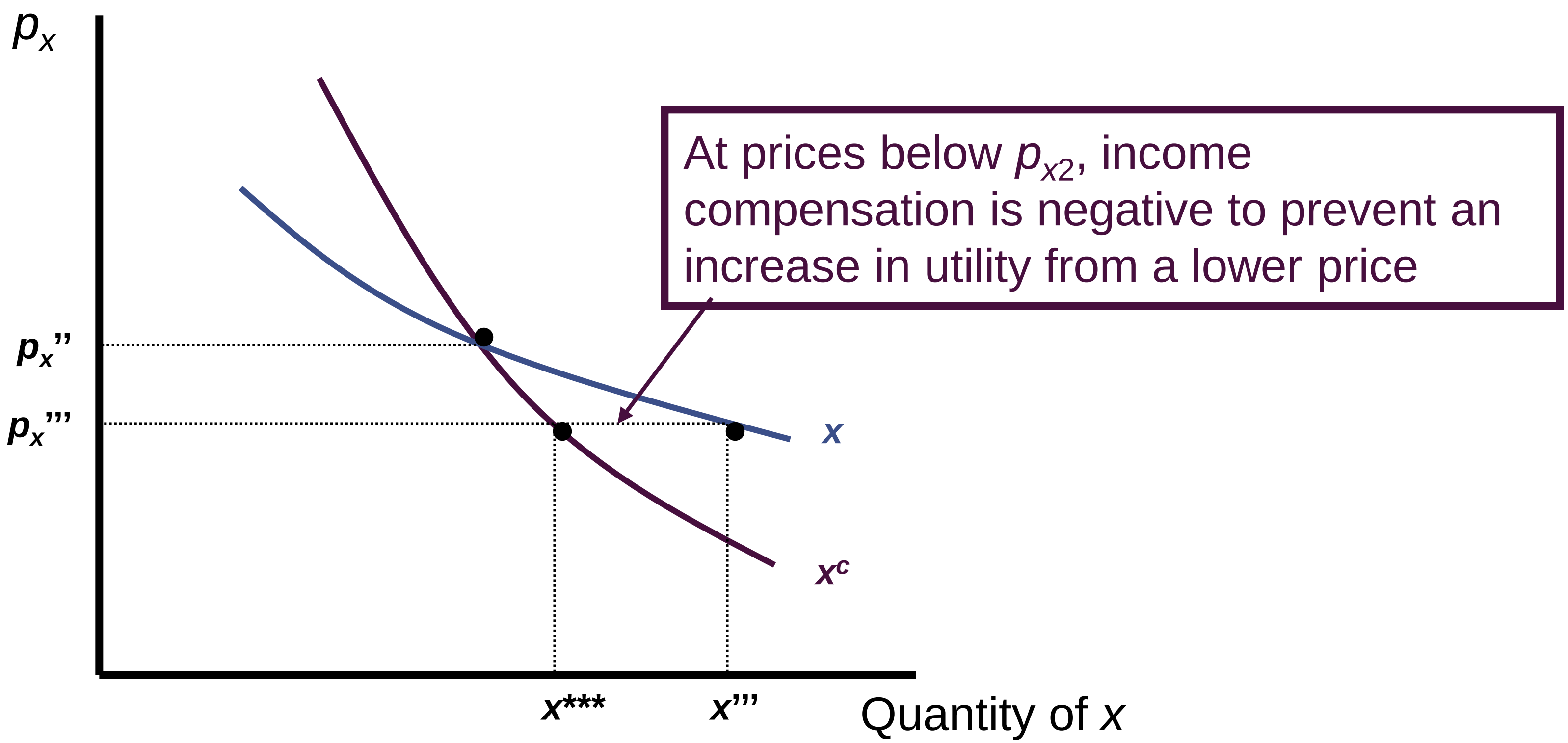
Compensated & Uncompensated Demand



Compensated & Uncompensated Demand



Compensated & Uncompensated Demand



Compensated & Uncompensated Demand

- For a normal good, the compensated demand curve is less responsive to price changes than is the uncompensated demand curve
  - the uncompensated demand curve reflects both income and substitution effects
  - the compensated demand curve reflects only substitution effects

Compensated Demand Functions

- Suppose that utility is given by

$$\text{utility} = U(x,y) = x^{0.5}y^{0.5}$$

- The Marshallian demand functions are

$$x = I/2p_x \qquad y = I/2p_y$$

- The indirect utility function is

$$\text{utility} = V(I, p_x, p_y) = \frac{I}{2p_x^{0.5}p_y^{0.5}}$$

Compensated Demand Functions

- To obtain the compensated demand functions, we can solve the indirect utility function for *I* （given a utility level V） and then substitute into the Marshallian demand functions

$$x = \frac{Vp_y^{0.5}}{p_x^{0.5}} \qquad y = \frac{Vp_x^{0.5}}{p_y^{0.5}}$$

Compensated Demand Functions

$$x = \frac{Vp_y^{0.5}}{p_x^{0.5}} \qquad y = \frac{Vp_x^{0.5}}{p_y^{0.5}}$$

- Demand now depends on utility (*V*) rather than income
- Increases in p_x reduce the amount of *x* demanded
  - only a substitution effect

A Mathematical Examination of a Change in Price

- Our goal is to examine how purchases of good x change when p_x changes

$$\partial x/\partial p_x$$

- Differentiation of the first-order conditions from utility maximization can be performed to solve for this derivative
- However, this approach is cumbersome and provides little economic insight

A Mathematical Examination of a Change in Price

- Instead, we will use an indirect approach
- Remember the expenditure function

$$\text{minimum expenditure} = E(p_x,p_y,U)$$

- Then, by definition

$$x^c\,(p_x,p_y,U) = x\,[p_x,p_y,E(p_x,p_y,U)]$$

  – quantity demanded is equal for both demand functions when income is exactly what is needed to attain the required utility level

A Mathematical Examination of a Change in Price

$$x^c(p_x,p_y,U) = x[p_x,p_y,E(p_x,p_y,U)]$$

- We can differentiate the compensated demand function and get

$$\frac{\partial x^c}{\partial p_x} = \frac{\partial x}{\partial p_x} + \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

A Mathematical Examination of a Change in Price

$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

- The first term is the slope of the compensated demand curve
  - the mathematical representation of the substitution effect

A Mathematical Examination of a Change in Price

$$\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - \frac{\partial x}{\partial E} \cdot \frac{\partial E}{\partial p_x}$$

- The second term measures the way in which changes in p_x affect the demand for x through changes in purchasing power
  - the mathematical representation of the income effect

The Slutsky Equation

- The substitution effect can be written as

$$\text{substitution effect} = \frac{\partial x^c}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}}$$

- The income effect can be written as

$$\text{income effect} = -\frac{\partial x}{\partial E}\cdot\frac{\partial E}{\partial p_x} = -\frac{\partial x}{\partial I}\cdot\frac{\partial E}{\partial p_x}$$

The Slutsky Equation

- Note that $\partial E/\partial p_x = x$
  - a \$1 increase in p_x raises necessary expenditures by x dollars
  - \$1 extra must be paid for each unit of x purchased

The Slutsky Equation

- The utility-maximization hypothesis shows that the substitution and income effects arising from a price change can be represented by

$$\frac{\partial x}{\partial p_x} = \text{substitution effect} + \text{income effect}$$

$$\frac{\partial x}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}} - x\frac{\partial x}{\partial I}$$

The Slutsky Equation

$$\frac{\partial x}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}} - x\frac{\partial x}{\partial I}$$

- The first term is the substitution effect
  - always negative as long as *MRS* is diminishing
  - the slope of the compensated demand curve must be negative

The Slutsky Equation

$$\frac{\partial x}{\partial p_x} = \left.\frac{\partial x}{\partial p_x}\right|_{U=\text{constant}} - x\frac{\partial x}{\partial I}$$

- The second term is the income effect
  - if *x* is a normal good, then $\partial x/\partial I > 0$
    - the entire income effect is negative
  - if *x* is an inferior good, then $\partial x/\partial I < 0$
    - the entire income effect is positive

A Slutsky Decomposition

- We can demonstrate the decomposition of a price effect using the Cobb-Douglas example studied earlier
- The Marshallian demand function for good *x* was

$$x(p_x, p_y, I) = \frac{0.5I}{p_x}$$

A Slutsky Decomposition

- The Hicksian (compensated) demand function for good *x* was

$$x^c(p_x, p_y, V) = \frac{V p_y^{0.5}}{p_x^{0.5}}$$

- The overall effect of a price change on the demand for *x* is

$$\frac{\partial x}{\partial p_x} = \frac{-0.5I}{p_x^2}$$

A Slutsky Decomposition

- This total effect is the sum of the two effects that Slutsky identified
- The substitution effect is found by differentiating the compensated demand function

$$\text{substitution effect} = \frac{\partial x^c}{\partial p_x} = \frac{-0.5Vp_y^{0.5}}{p_x^{1.5}}$$

A Slutsky Decomposition

- We can substitute in for the indirect utility function (V)

$$\text{substitution effect} = \frac{-0.5(0.5Ip_x^{-0.5}p_y^{-0.5})p_y^{0.5}}{p_x^{1.5}} = \frac{-0.25I}{p_x^2}$$

A Slutsky Decomposition

- Calculation of the income effect is easier

$$\text{income effect} = -x\frac{\partial x}{\partial I} = -\left[\frac{0.5I}{p_x}\right]\cdot\frac{0.5}{p_x} = -\frac{0.25I}{p_x^2}$$

- Interestingly, the substitution and income effects are exactly the same size

Marshallian Demand Elasticities

- Most of the commonly used demand elasticities are derived from the Marshallian demand function $x(p_x,p_y,I)$
- Price elasticity of demand ($e_{x,px}$)

$$e_{x,p_x} = \frac{\Delta x / x}{\Delta p_x / p_x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x}$$

Marshallian Demand Elasticities

- Income elasticity of demand ($e_{x,I}$)

$$e_{x,I} = \frac{\Delta x / x}{\Delta I / I} = \frac{\partial x}{\partial I} \cdot \frac{I}{x}$$

- Cross-price elasticity of demand ($e_{x,py}$)

$$e_{x,p_y} = \frac{\Delta x / x}{\Delta p_y / p_y} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x}$$

Price Elasticity of Demand

- The own price elasticity of demand is always negative
  - the only exception is Giffen's paradox
- The size of the elasticity is important
  - if $e_{x,px} < -1$, demand is elastic
  - if $e_{x,px} > -1$, demand is inelastic
  - if $e_{x,px} = -1$, demand is unit elastic

Price Elasticity and Total Spending

- Total spending on *x* is equal to

$$\text{total spending} = p_x x$$

- Using elasticity, we can determine how total spending changes when the price of *x* changes

$$\frac{\partial(p_x x)}{\partial p_x} = p_x \cdot \frac{\partial x}{\partial p_x} + x = x[e_{x,p_x} + 1]$$

Price Elasticity and Total Spending

$$\frac{\partial(p_x x)}{\partial p_x} = p_x \cdot \frac{\partial x}{\partial p_x} + x = x[e_{x,p_x} + 1]$$

- The sign of this derivative depends on whether $e_{x,px}$ is greater or less than -1
  - if $e_{x,px} > -1$, demand is inelastic and price and total spending move in the same direction
  - if $e_{x,px} < -1$, demand is elastic and price and total spending move in opposite directions

Compensated Price Elasticities

- It is also useful to define elasticities based on the compensated demand function

Compensated Price Elasticities

- If the compensated demand function is

$$x^c = x^c(p_x,p_y,U)$$

  we can calculate
  - compensated own price elasticity of demand ($e_{x^c,px}$)
  - compensated cross-price elasticity of demand ($e_{x^c,py}$)

Compensated Price Elasticities

- The compensated own price elasticity of demand ($e_x{}^c{}_{,px}$) is

$$e^c_{x,p_x} = \frac{\Delta x^c / x^c}{\Delta p_x / p_x} = \frac{\partial x^c}{\partial p_x} \cdot \frac{p_x}{x^c}$$

- The compensated cross-price elasticity of demand ($e_x{}^c{}_{,py}$) is

$$e^c_{x,p_y} = \frac{\Delta x^c / x^c}{\Delta p_y / p_y} = \frac{\partial x^c}{\partial p_y} \cdot \frac{p_y}{x^c}$$

Compensated Price Elasticities

- The relationship between Marshallian and compensated price elasticities can be shown using the Slutsky equation

$$\frac{p_x}{x} \cdot \frac{\partial x}{\partial p_x} = e_{x,p_x} = \frac{p_x}{x^c} \cdot \frac{\partial x^c}{\partial p_x} - \frac{p_x}{x} \cdot x \cdot \frac{\partial x}{\partial I}$$

- If $s_x = p_x x / I$, then

$$e_{x,p_x} = e^c_{x,p_x} - s_x e_{x,I}$$

Compensated Price Elasticities

- The Slutsky equation shows that the compensated and uncompensated price elasticities will be similar if
  - the share of income devoted to *x* is small
  - the income elasticity of *x* is small

Homogeneity

- Demand functions are homogeneous of degree zero in all prices and income
- Euler's theorem for homogenous functions shows that

$$0 = p_x \cdot \frac{\partial x}{\partial p_x} + p_y \cdot \frac{\partial x}{\partial p_y} + I \cdot \frac{\partial x}{\partial I}$$

Homogeneity

- Dividing by *x*, we get

$$0 = e_{x,p_x} + e_{x,p_y} + e_{x,I}$$

- Any proportional change in all prices and income will leave the quantity of *x* demanded unchanged

Engel Aggregation

- Engel's law suggests that the income elasticity of demand for food items is less than one
  - this implies that the income elasticity of demand for all nonfood items must be greater than one

Engel Aggregation

- We can see this by differentiating the budget constraint with respect to income (treating prices as constant)

$$1 = p_x \cdot \frac{\partial x}{\partial I} + p_y \cdot \frac{\partial y}{\partial I}$$

$$1 = p_x \cdot \frac{\partial x}{\partial I} \cdot \frac{xI}{xI} + p_y \cdot \frac{\partial y}{\partial I} \cdot \frac{yI}{yI} = s_x e_{x,I} + s_y e_{y,I}$$

Cournot Aggregation

- The size of the cross-price effect of a change in the price of x on the quantity of y consumed is restricted because of the budget constraint
- We can demonstrate this by differentiating the budget constraint with respect to p_x

Cournot Aggregation

$$\frac{\partial I}{\partial p_x} = 0 = p_x \cdot \frac{\partial x}{\partial p_x} + x + p_y \cdot \frac{\partial y}{\partial p_x}$$

$$0 = p_x \cdot \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{I} \cdot \frac{x}{x} + x \cdot \frac{p_x}{I} + p_y \cdot \frac{\partial y}{\partial p_x} \cdot \frac{p_x}{I} \cdot \frac{y}{y}$$

$$0 = s_x e_{x,p_x} + s_x + s_y e_{y,p_x}$$

$$s_x e_{x,p_x} + s_y e_{y,p_x} = -s_x$$

Demand Elasticities

- The Cobb-Douglas utility function is

$$U(x,y) = x^{\alpha}y^{\beta} \qquad (\alpha+\beta=1)$$

- The demand functions for *x* and *y* are

$$x = \frac{\alpha I}{p_x} \qquad\qquad y = \frac{\beta I}{p_y}$$

Demand Elasticities

- Calculating the elasticities, we get

$$e_{x,p_x} = \frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x} = -\frac{\alpha I}{p_x^2} \cdot \frac{p_x}{\left(\frac{\alpha I}{p_x}\right)} = -1$$

$$e_{x,p_y} = \frac{\partial x}{\partial p_y} \cdot \frac{p_y}{x} = 0 \cdot \frac{p_y}{x} = 0$$

$$e_{x,I} = \frac{\partial x}{\partial I} \cdot \frac{I}{x} = \frac{\alpha}{p_x} \cdot \frac{I}{\left(\frac{\alpha I}{p_x}\right)} = 1$$

Demand Elasticities

- We can also show
  - homogeneity

$$e_{x,p_x} + e_{x,p_y} + e_{x,I} = -1 + 0 + 1 = 0$$

  - Engel aggregation

$$s_x e_{x,I} + s_y e_{y,I} = \alpha \cdot 1 + \beta \cdot 1 = \alpha + \beta = 1$$

  - Cournot aggregation

$$s_x e_{x,p_x} + s_y e_{y,p_x} = \alpha \cdot (-1) + \beta \cdot 0 = -\alpha = -s_x$$

Demand Elasticities

- We can also use the Slutsky equation to derive the compensated price elasticity

$$e^{c}_{x,p_x} = e_{x,p_x} + s_x e_{x,I} = -1 + \alpha(1) = \alpha - 1 = -\beta$$

- The compensated price elasticity depends on how important other goods (*y*) are in the utility function

Demand Elasticities

- The CES utility function (with $\sigma = 2$, $\delta = 5$) is

$$U(x,y) = x^{0.5} + y^{0.5}$$

- The demand functions for x and y are

$$x = \frac{I}{p_x(1 + p_x p_y^{-1})} \qquad y = \frac{I}{p_y(1 + p_x^{-1} p_y)}$$

Demand Elasticities

- We will use the "share elasticity" to derive the own price elasticity

$$e_{s_x, p_x} = \frac{\partial s_x}{\partial p_x} \cdot \frac{p_x}{s_x} = 1 + e_{x, p_x}$$

- In this case,

$$s_x = \frac{p_x x}{I} = \frac{1}{1 + p_x p_y^{-1}}$$

Demand Elasticities

- Thus, the share elasticity is given by

$$e_{s_x, p_x} = \frac{\partial s_x}{\partial p_x} \cdot \frac{p_x}{s_x} = \frac{-p_y^{-1}}{(1 + p_x p_y^{-1})^2} \cdot \frac{p_x}{(1 + p_x p_y^{-1})^{-1}} = \frac{-p_x p_y^{-1}}{1 + p_x p_y^{-1}}$$

- Therefore, if we let $p_x = p_y$

$$e_{x, p_x} = e_{s_x, p_x} - 1 = \frac{-1}{1+1} - 1 = -1.5$$

Demand Elasticities

- The CES utility function (with $\sigma = 0.5$, $\delta = -1$) is

$$U(x,y) = -x^{-1} - y^{-1}$$

- The share of good *x* is

$$s_x = \frac{p_x x}{I} = \frac{1}{1 + p_y^{0.5} p_x^{-0.5}}$$

Demand Elasticities

- Thus, the share elasticity is given by

$$e_{s_x,p_x} = \frac{\partial s_x}{\partial p_x} \cdot \frac{p_x}{s_x} = \frac{0.5 p_y^{0.5} p_x^{-1.5}}{(1 + p_y^{0.5} p_x^{-0.5})^2} \cdot \frac{p_x}{(1 + p_y^{0.5} p_x^{-0.5})^{-1}}$$

$$= \frac{0.5 p_y^{0.5} p_x^{-0.5}}{1 + p_y^{0.5} p_x^{-0.5}}$$

- Again, if we let $p_x = p_y$

$$e_{x,p_x} = e_{s_x,p_x} - 1 = \frac{0.5}{2} - 1 = -0.75$$

Consumer Surplus 消费者剩余

- An important problem in welfare economics is to devise a monetary measure of the gains and losses that individuals experience when prices change

Consumer Welfare

- One way to evaluate the welfare cost of a price increase (from p_x^0 to p_x^1) would be to compare the expenditures required to achieve U_0 under these two situations

$$\text{expenditure at } p_x^0 = E_0 = E(p_x^0, p_y, U_0)$$

$$\text{expenditure at } p_x^1 = E_1 = E(p_x^1, p_y, U_0)$$

Consumer Welfare

- In order to compensate for the price rise, this person would require a compensating variation (*CV*) of

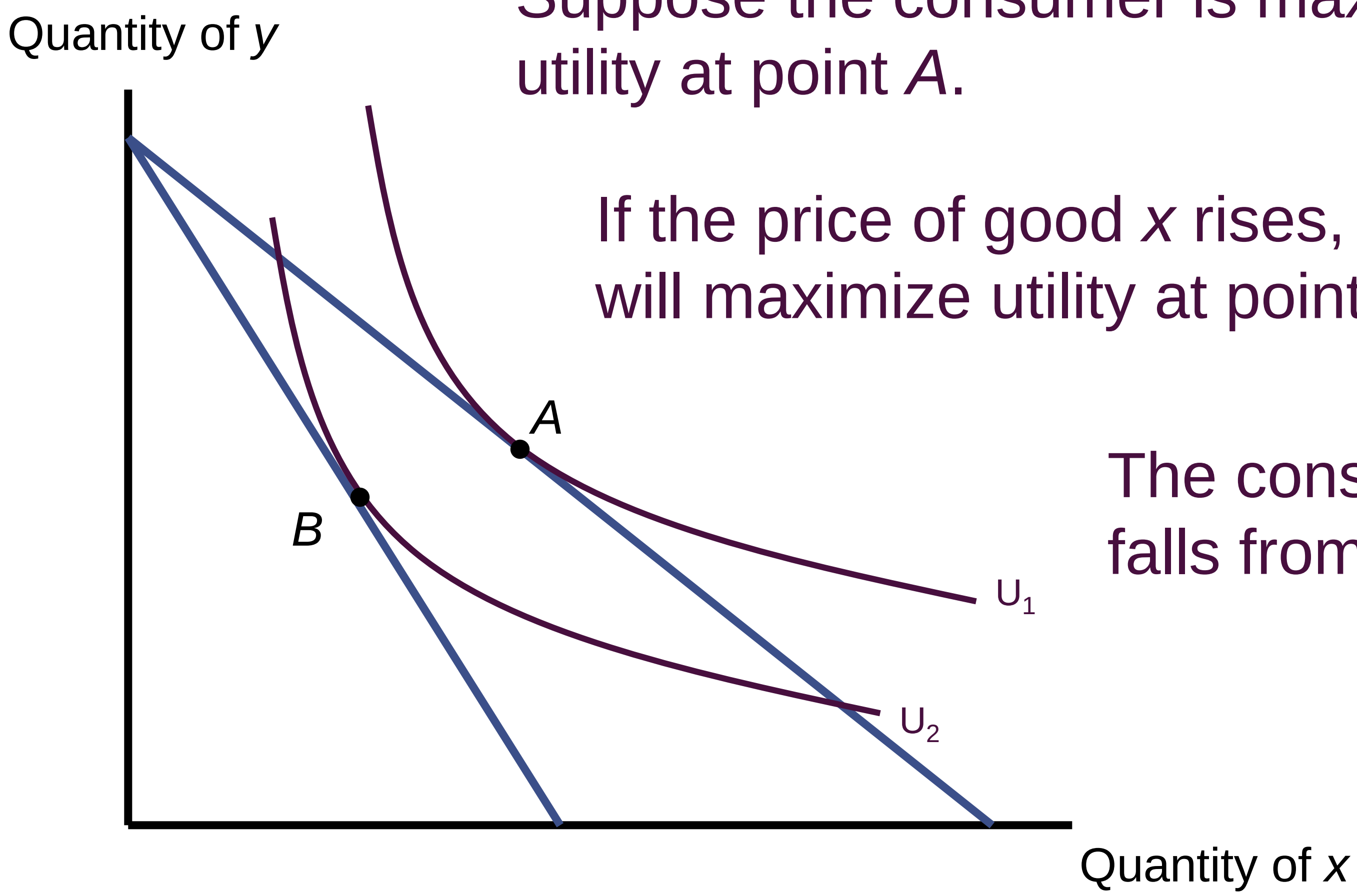
$$CV = E(p_x^{\mathbf{1}}, p_y, U_0) - E(p_x^{\mathbf{0}}, p_y, U_0)$$

Consumer Welfare

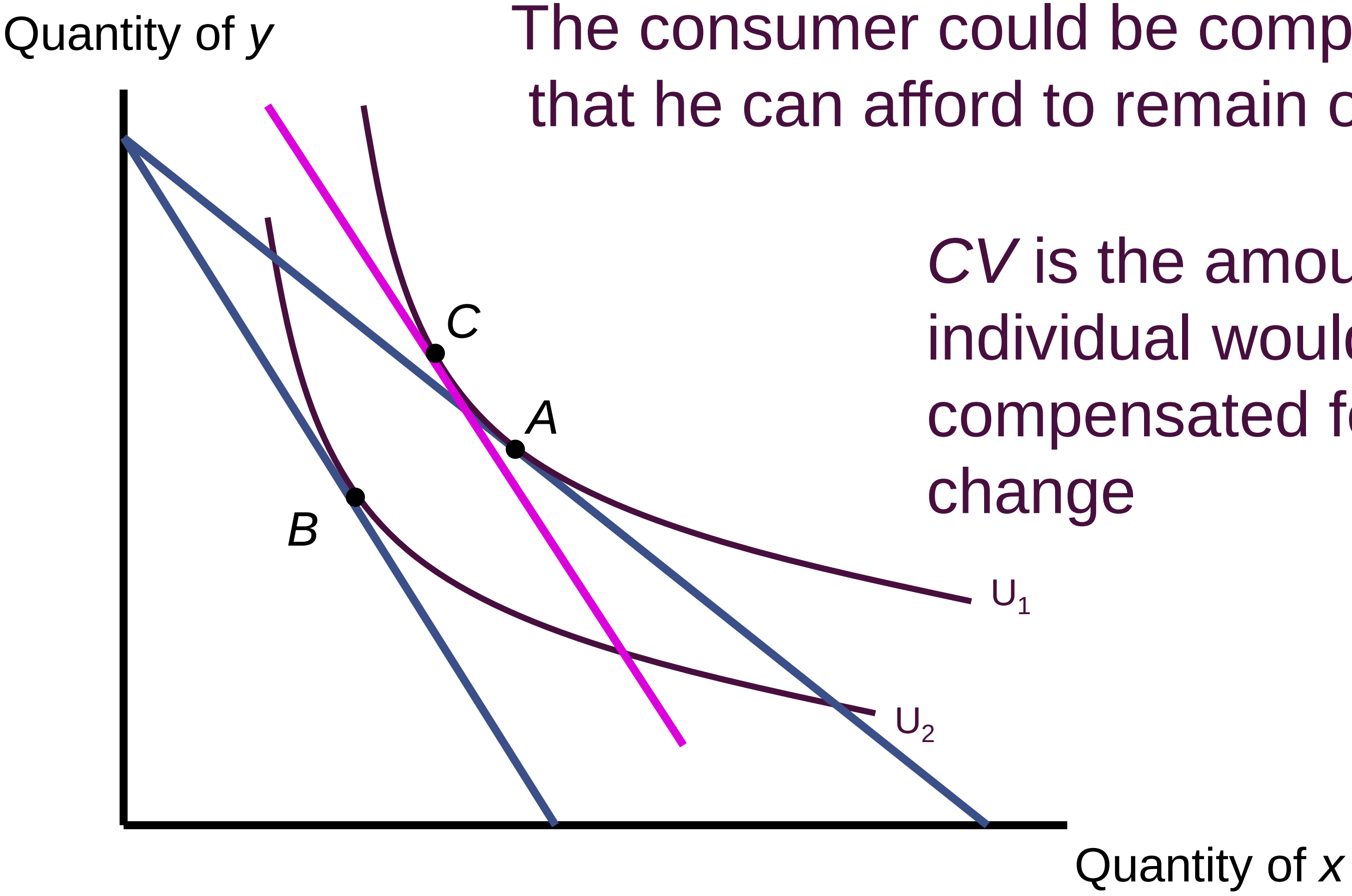
Suppose the consumer is maximizing utility at point *A*.

If the price of good *x* rises, the consumer will maximize utility at point *B*.

The consumer's utility falls from U_1 to U_2



Consumer Welfare



The consumer could be compensated so that he can afford to remain on U_1

CV is the amount that the individual would need to be compensated for price change

Consumer Welfare

- The derivative of the expenditure function with respect to p_x is the compensated demand function

$$\frac{\partial E(p_x, p_y, U_0)}{\partial p_x} = x^c(p_x, p_y, U_0)$$

Consumer Welfare

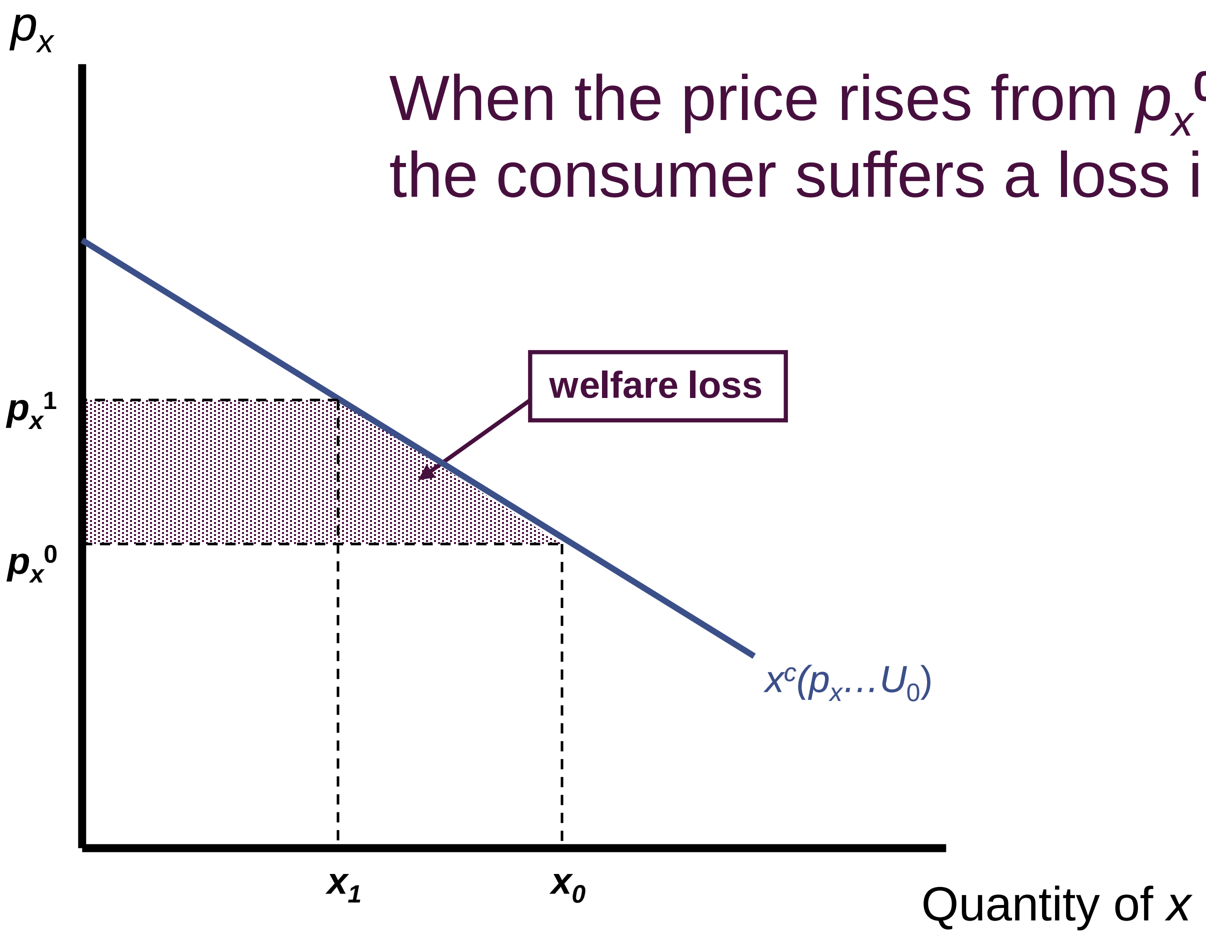
- The amount of *CV* required can be found by integrating across a sequence of small increments to price from p_x^0 to p_x^1

$$CV = \int_{p_x^0}^{p_x^1} dE = \int_{p_x^0}^{p_x^1} x^c(p_x, p_y, U_0)\,dp_x$$

  – this integral is the area to the left of the compensated demand curve between $p_x^{\mathbf{0}}$ and $p_x^{\mathbf{1}}$

Consumer Welfare

When the price rises from p_x^0 to p_x^1, the consumer suffers a loss in welfare



Consumer Welfare

- Because a price change generally involves both income and substitution effects, it is unclear which compensated demand curve should be used
- Do we use the compensated demand curve for the original target utility (U_0) or the new level of utility after the price change (U_1)?

The Consumer Surplus Concept

- Another way to look at this issue is to ask how much the person would be willing to pay for the right to consume all of this good that he wanted at the market price of p_x^0

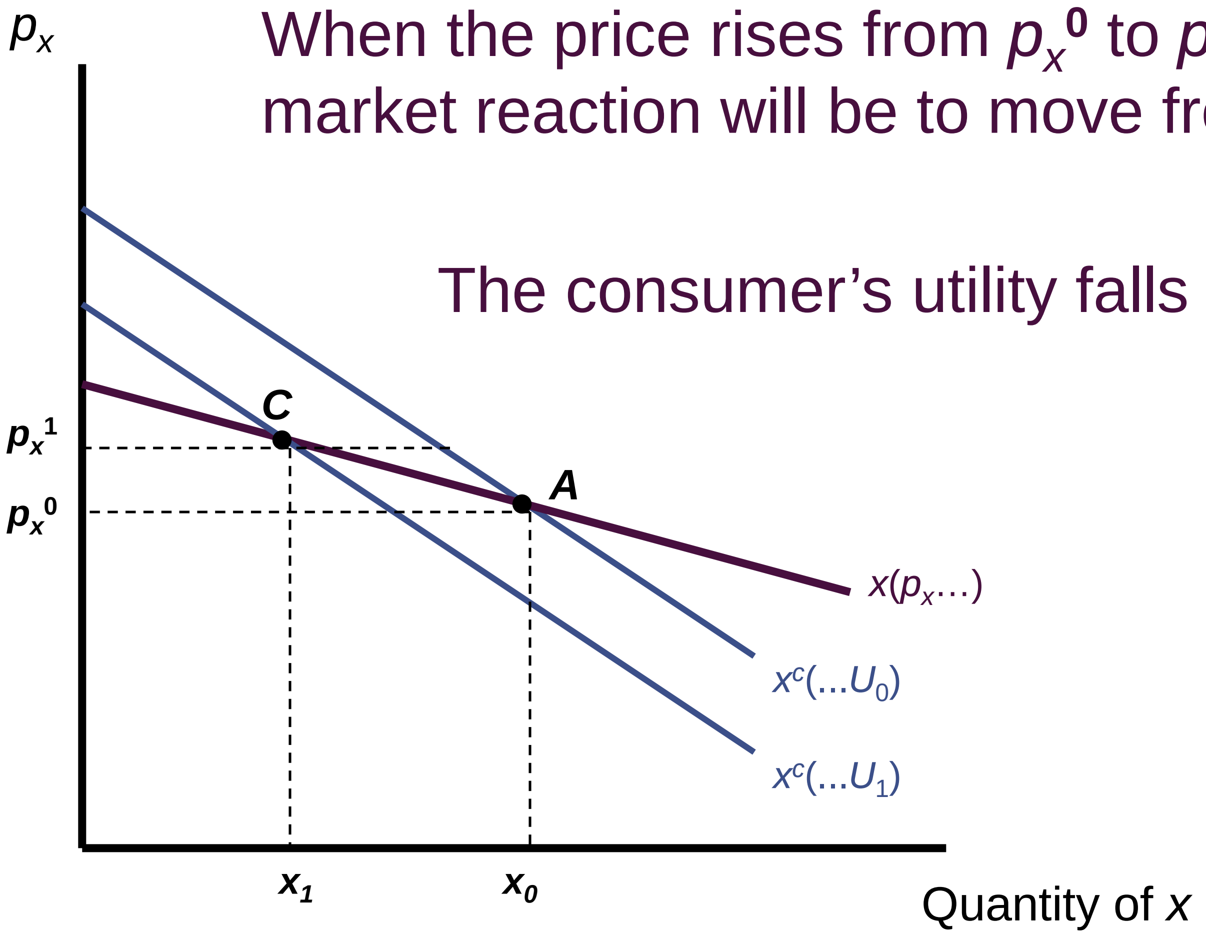
The Consumer Surplus Concept

- The area below the compensated demand curve and above the market price is called consumer surplus
  - the extra benefit the person receives by being able to make market transactions at the prevailing market price

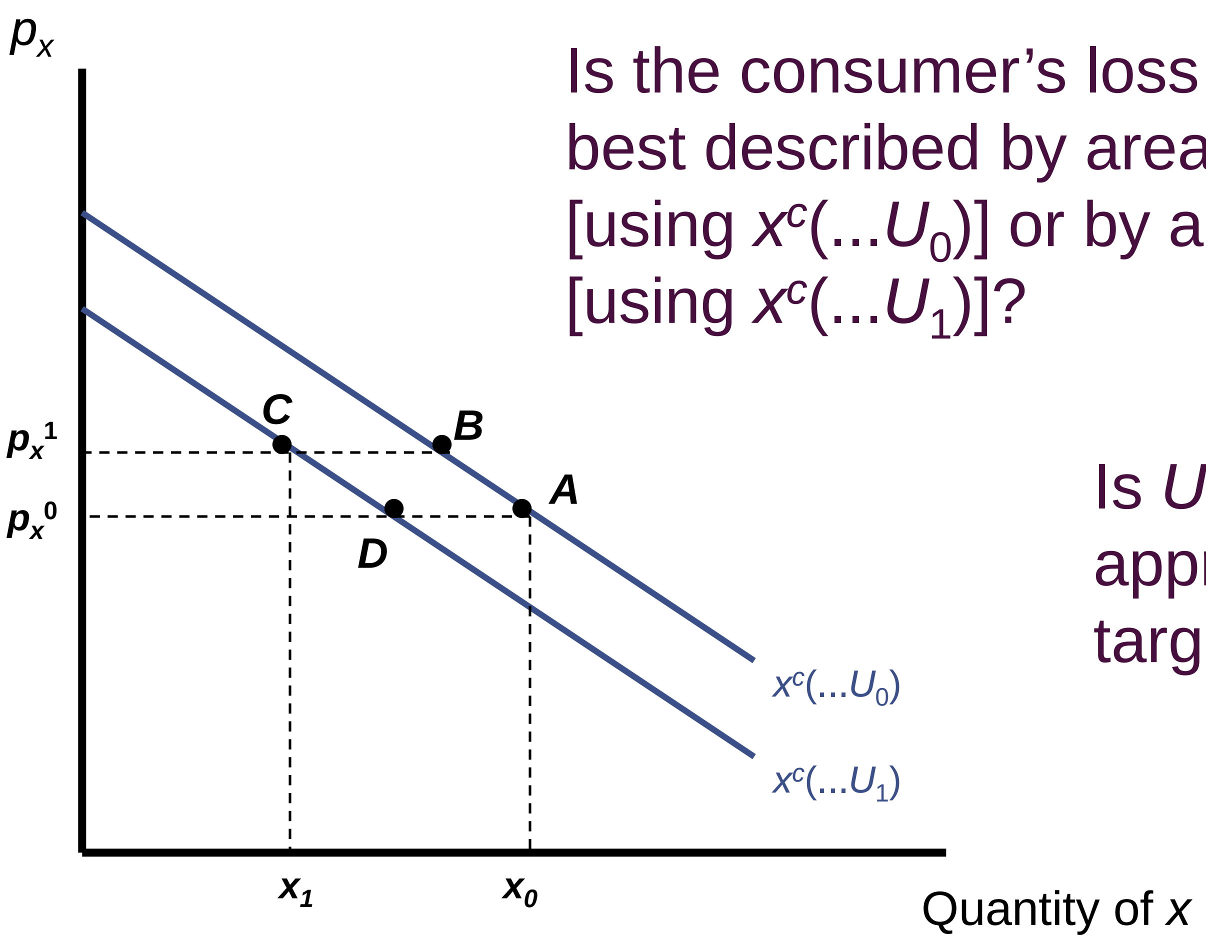
Consumer Welfare

When the price rises from $p_x^{\mathbf{0}}$ to $p_x^{\mathbf{1}}$, the actual market reaction will be to move from ***A*** to ***C***

The consumer's utility falls from U_0 to U_1



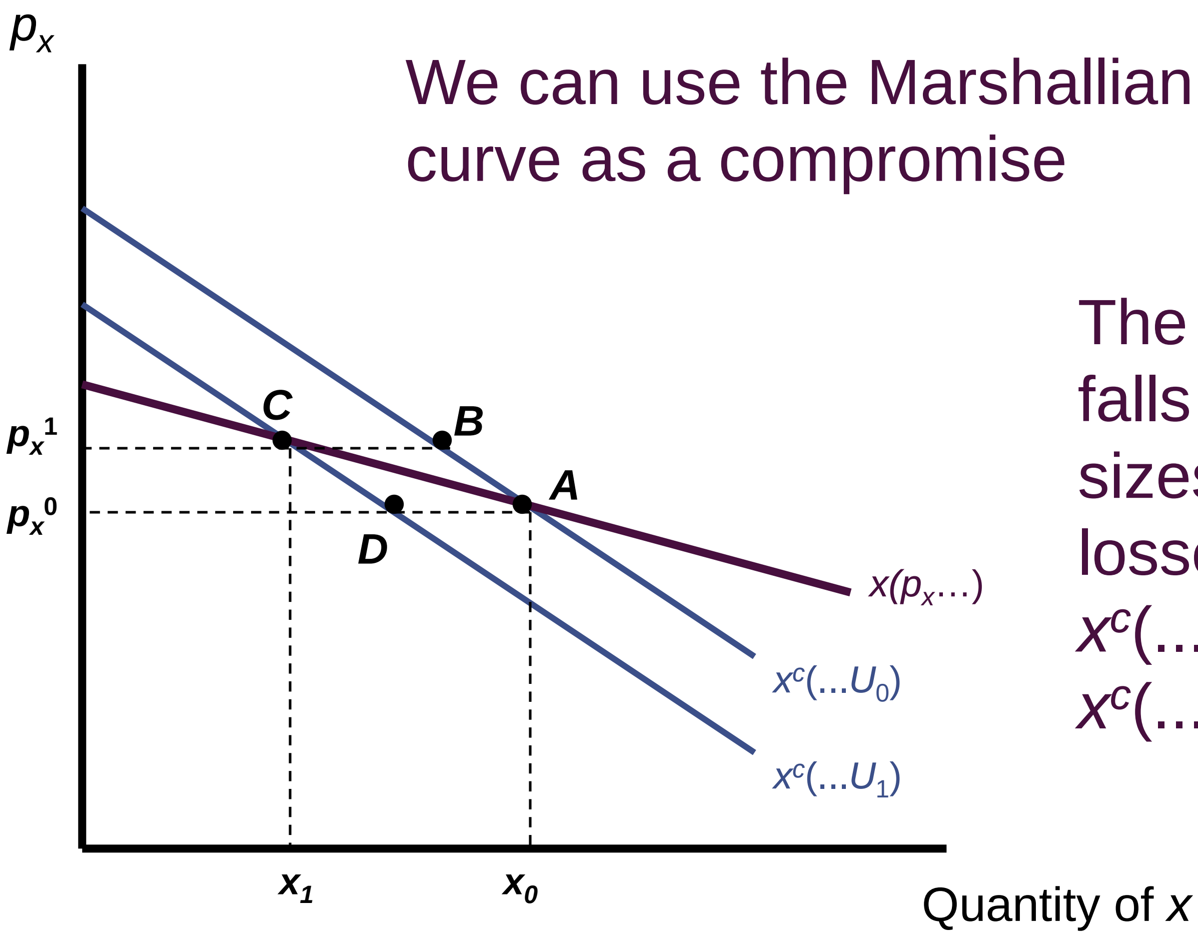
Consumer Welfare



Is the consumer's loss in welfare best described by area $\boldsymbol{p_x^1 B A p_x^0}$ [using $x^c(\ldots U_0)$] or by area $\boldsymbol{p_x^1 C D p_x^0}$ [using $x^c(\ldots U_1)$]?

Is U_0 or U_1 the appropriate utility target?

Consumer Welfare



We can use the Marshallian demand curve as a compromise

The area $\boldsymbol{p_x^1CAp_x^0}$ falls between the sizes of the welfare losses defined by $x^c(\ldots U_0)$ and $x^c(\ldots U_1)$

Consumer Surplus

- We will define consumer surplus as the area below the Marshallian demand curve and above price
  - shows what an individual would pay for the right to make voluntary transactions at this price
  - changes in consumer surplus measure the welfare effects of price changes

Example: Welfare Loss from a Price Increase (skipped)

- Suppose that the compensated demand function for *x* is given by

$$x^c(p_x,p_y,V) = \frac{Vp_y^{0.5}}{p_x^{0.5}}$$

- The welfare cost of a price increase from $p_x = 1$ to $p_x = 4$ is given by

$$CV = \int_1^4 Vp_y^{0.5}p_x^{-0.5} = 2Vp_y^{0.5}p_x^{0.5}\Big|_{p_x=1}^{p_x=4}$$

Welfare Loss from a Price Increase

- If we assume that $V = 2$ and $p_y = 2$,

$$CV = 2 \cdot 2 \cdot 2 \cdot (4)^{0.5} - 2 \cdot 2 \cdot 2 \cdot (1)^{0.5} = 8$$

- If we assume that the utility level (V) falls to 1 after the price increase (and used this level to calculate welfare loss),

$$CV = 1 \cdot 2 \cdot 2 \cdot (4)^{0.5} - 1 \cdot 2 \cdot 2 \cdot (1)^{0.5} = 4$$

Welfare Loss from Price Increase

- Suppose that we use the Marshallian demand function instead

$$x(p_x, p_y, I) = 0.5 I p_x^{-1}$$

- The welfare loss from a price increase from $p_x = 1$ to $p_x = 4$ is given by

$$Loss = \int_1^4 0.5 I p_x^{-1} dp_x = 0.5 I \ln p_x \Big|_{p_x=1}^{p_x=4}$$

Welfare Loss from a Price Increase

- If income (*I*) is equal to 8,

$$\text{loss} = 4\ln(4) - 4\ln(1) = 4\ln(4) = 4(1.39) = 5.55$$

  – this computed loss from the Marshallian demand function is a compromise between the two amounts computed using the compensated demand functions

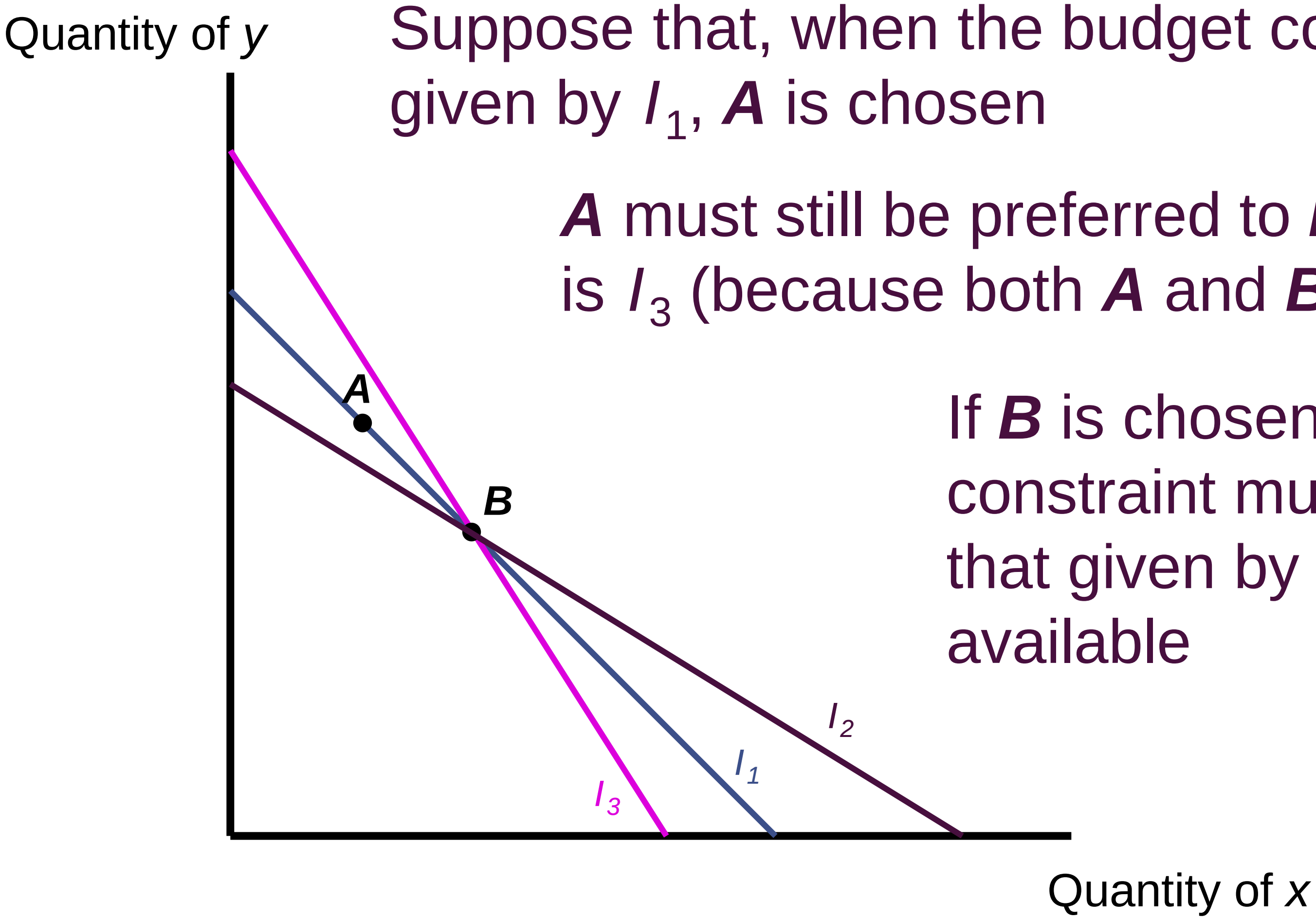
Revealed Preference and the Substitution Effect

- The theory of revealed preference was proposed by Paul Samuelson in the late 1940s
- The theory defines a principle of rationality based on observed behavior and then uses it to approximate an individual's utility function

Revealed Preference and the Substitution Effect

- Consider two bundles of goods: ***A*** and ***B***
- If the individual can afford to purchase either bundle but chooses ***A***, we say that ***A*** had been revealed preferred to ***B***
- Under any other price-income arrangement, ***B*** can never be revealed preferred to ***A***

Revealed Preference and the Substitution Effect



Suppose that, when the budget constraint is given by I_1, ***A*** is chosen

A must still be preferred to ***B*** when income is I_3 (because both ***A*** and ***B*** are available)

If ***B*** is chosen, the budget constraint must be similar to that given by I_2 where ***A*** is not available

Negativity of the Substitution Effect

- Suppose that an individual is indifferent between two bundles: ***C*** and ***D***
- Let p_x^C, p_y^C be the prices at which bundle ***C*** is chosen
- Let p_x^D, p_y^D be the prices at which bundle ***D*** is chosen

Negativity of the Substitution Effect

- Since the individual is indifferent between ***C*** and ***D***
  - When ***C*** is chosen, ***D*** must cost at least as much as ***C***

$$p_x^C x_C + p_y^C y_C \leq p_x^C x_D + p_y^C y_D$$

  - When ***D*** is chosen, ***C*** must cost at least as much as ***D***

$$p_x^D x_D + p_y^D y_D \leq p_x^D x_C + p_y^D y_C$$

Negativity of the Substitution Effect

- Rearranging, we get

$$p_x^C(x_C - x_D) + p_y^C(y_C - y_D) \leq 0$$

$$p_x^D(x_D - x_C) + p_y^D(y_D - y_C) \leq 0$$

- Adding these together, we get

$$(p_x^C - p_x^D)(x_C - x_D) + (p_y^C - p_y^D)(y_C - y_D) \leq 0$$

Negativity of the Substitution Effect

- Suppose that only the price of x changes ($p_y^C = p_y^D$)

$$(p_x^C - p_x^D)(x_C - x_D) \leq 0$$

- This implies that price and quantity move in opposite direction when utility is held constant
  - the substitution effect is negative

Mathematical Generalization

- If, at prices p_i^0 bundle $\boldsymbol{x}_i^0$ is chosen instead of bundle $\boldsymbol{x}_i^1$ (and bundle $\boldsymbol{x}_i^1$ is affordable), then

$$\sum_{i=1}^{n} p_i^0 x_i^0 \geq \sum_{i=1}^{n} p_i^0 x_i^1$$

- Bundle ***0*** has been "revealed preferred" to bundle ***1***

Mathematical Generalization

- Consequently, at prices that prevail when bundle ***1*** is chosen (p_i^1), then

$$\sum_{i=1}^{n} p_i^1 x_i^0 > \sum_{i=1}^{n} p_i^1 x_i^1$$

- Bundle ***0*** must be more expensive than bundle ***1***

Strong Axiom of Revealed Preference

- If commodity bundle ***0*** is revealed preferred to bundle ***1***, and if bundle ***1*** is revealed preferred to bundle ***2***, and if bundle ***2*** is revealed preferred to bundle ***3***,…,and if bundle ***K-1*** is revealed preferred to bundle ***K***, then bundle ***K*** <u>cannot</u> be revealed preferred to bundle ***0***

Important Points to Note:

- Proportional changes in all prices and income do not shift the individual's budget constraint and therefore do not alter the quantities of goods chosen
  - demand functions are homogeneous of degree zero in all prices and income

Important Points to Note:

- When purchasing power changes (income changes but prices remain the same), budget constraints shift
  - for normal goods, an increase in income means that more is purchased
  - for inferior goods, an increase in income means that less is purchased

Important Points to Note:

- A fall in the price of a good causes substitution and income effects
  - for a normal good, both effects cause more of the good to be purchased
  - for inferior goods, substitution and income effects work in opposite directions
    - no unambiguous prediction is possible

Important Points to Note:

- A rise in the price of a good also causes income and substitution effects
  - for normal goods, less will be demanded
  - for inferior goods, the net result is ambiguous

Important Points to Note:

- The Marshallian demand curve summarizes the total quantity of a good demanded at each possible price
  - changes in price prompt movements along the curve
  - changes in income, prices of other goods, or preferences may cause the demand curve to shift

Important Points to Note:

- Compensated demand curves illustrate movements along a given indifference curve for alternative prices
  - they are constructed by holding utility constant and exhibit only the substitution effects from a price change
  - their slope is unambiguously negative (or zero)

Important Points to Note:

- Demand elasticities are often used in empirical work to summarize how individuals react to changes in prices and income
  - the most important is the price elasticity of demand
    - measures the proportionate change in quantity in response to a 1 percent change in price

Important Points to Note:

- There are many relationships among demand elasticities
  - own-price elasticities determine how a price change affects total spending on a good
  - substitution and income effects can be summarized by the Slutsky equation
  - various aggregation results hold among elasticities

Important Points to Note:

- Welfare effects of price changes can be measured by changing areas below either compensated or ordinary demand curves
  - such changes affect the size of the consumer surplus that individuals receive by being able to make market transactions

Important Points to Note:

- The negativity of the substitution effect is one of the most basic findings of demand theory
  - this result can be shown using revealed preference theory and does not necessarily require assuming the existence of a utility function