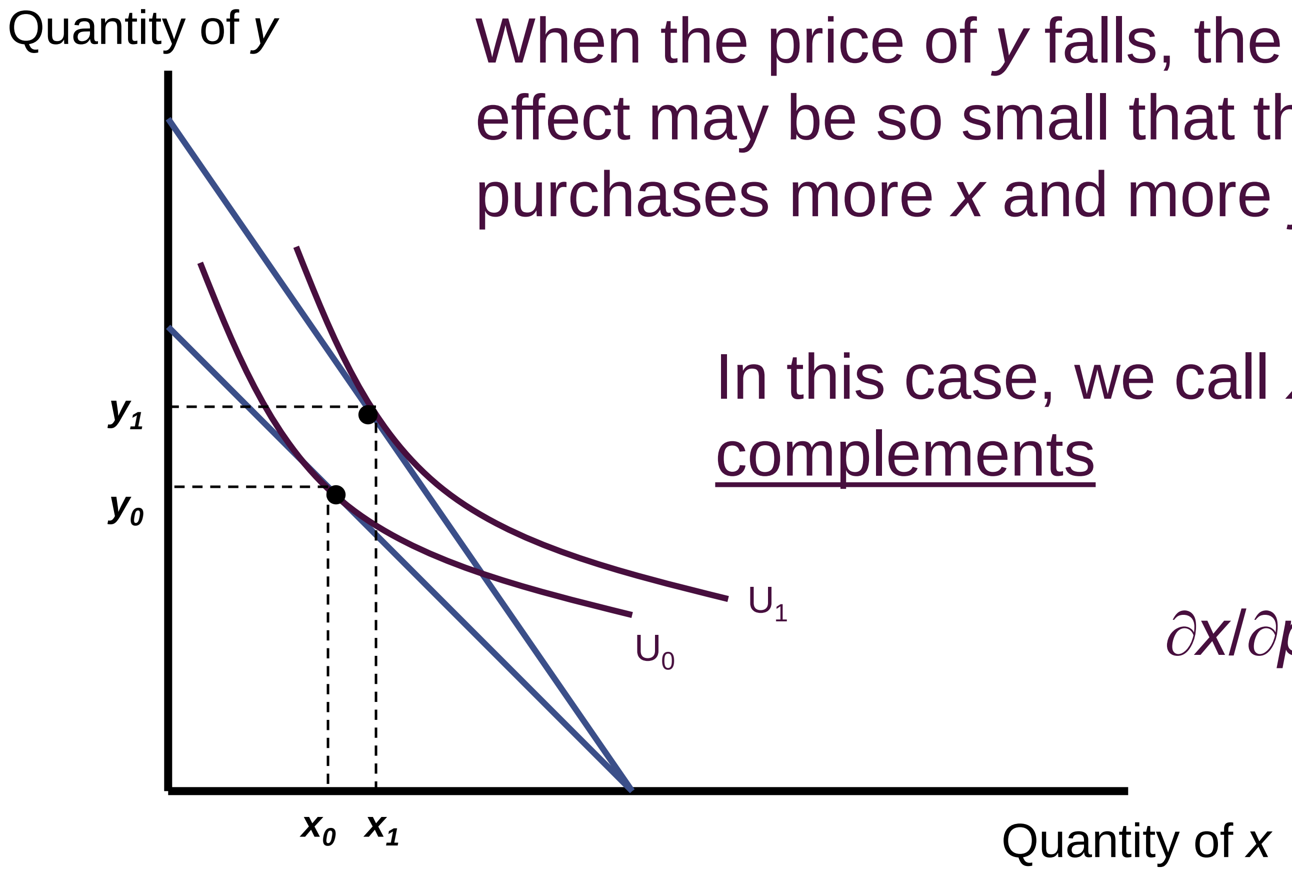
Chapter 6

DEMAND RELATIONSHIPS AMONG GOODS

The Two-Good Case

- The types of relationships that can occur when there are only two goods are limited
- But this case can be illustrated with two-dimensional graphs

Gross Complements

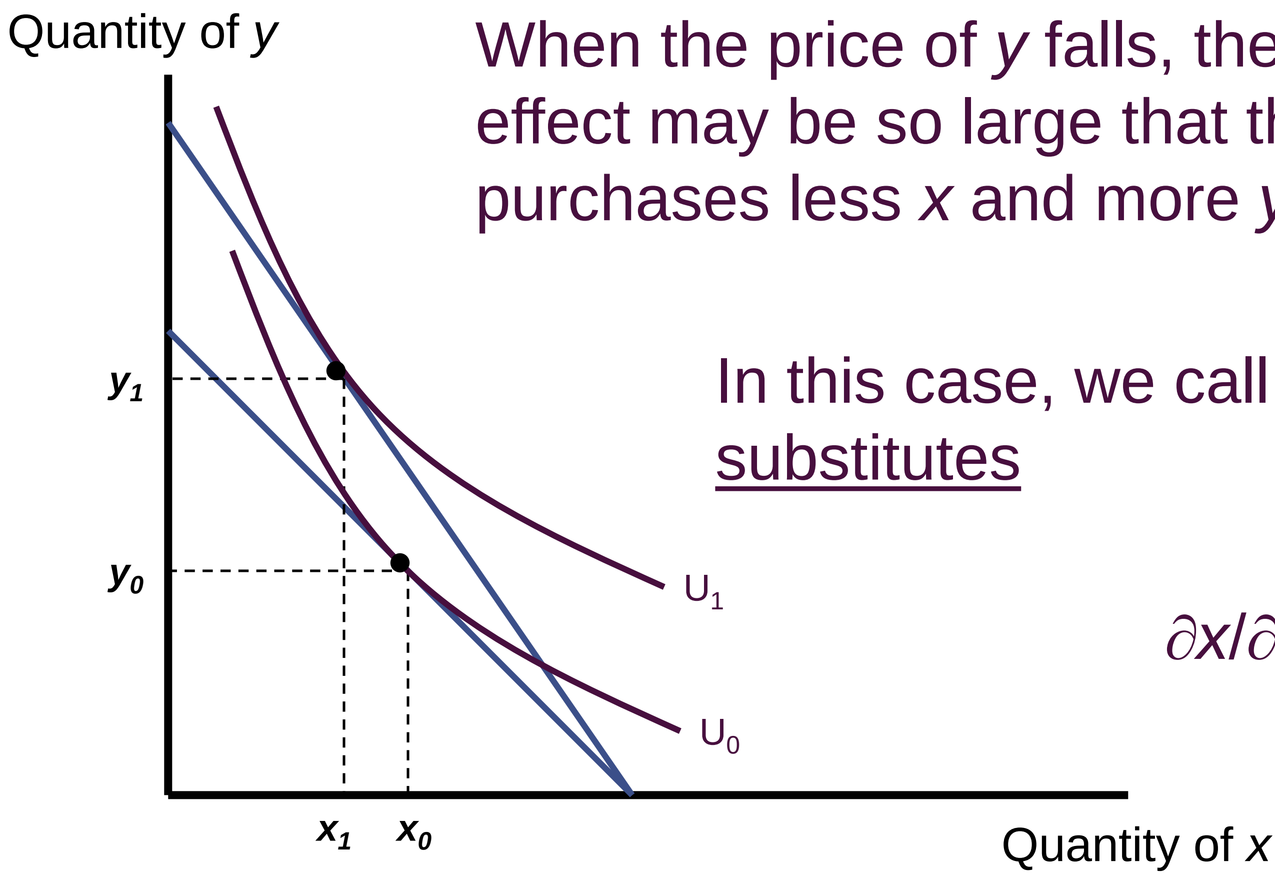


When the price of *y* falls, the substitution effect may be so small that the consumer purchases more *x* and more *y*

In this case, we call *x* and *y* <u>gross complements</u>

$$\partial x/\partial p_y < 0$$

Gross Substitutes



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$\partial x / \partial p_y > 0$

A Mathematical Treatment

- The change in *x* caused by changes in p_y can be shown by a Slutsky-type equation

$$\frac{\partial x}{\partial p_y} = \underbrace{\left.\frac{\partial x}{\partial p_y}\right|_{U=\text{constant}}}_{\text{substitution effect (+)}} \underbrace{- \; y \frac{\partial x}{\partial I}}_{\text{income effect (-) if } x \text{ is normal}}$$

substitution effect (+)

income effect (-) if *x* is normal

combined effect (ambiguous)

Substitutes (替代品) and Complements （互补品）

- For the case of many goods, we can generalize the Slutsky analysis

$$\frac{\partial x_i}{\partial p_j} = \left.\frac{\partial x_i}{\partial p_j}\right|_{U=\text{constant}} - x_j \frac{\partial x_i}{\partial I}$$

for any *i* or *j*

  – this implies that the change in the price of any good induces income and substitution effects that may change the quantity of every good demanded

Substitutes and Complements

- Two goods are substitutes if one good may replace the other in use
  - examples: tea & coffee, butter & margarine
- Two goods are complements if they are used together
  - examples: coffee & cream, fish & chips

Gross（总） Substitutes and Complements

- The concepts of gross substitutes and complements include both substitution and income effects
  - two goods are gross substitutes if

$$\partial x_i / \partial p_j > 0$$

  - two goods are gross complements if

$$\partial x_i / \partial p_j < 0$$

Asymmetry of the Gross Definitions

- One undesirable characteristic of the gross definitions of substitutes and complements is that they are not symmetric
- It is possible for x_1 to be a substitute for x_2 and at the same time for x_2 to be a complement of x_1

Asymmetry of the Gross Definitions

- Suppose that the utility function for two goods is given by

$$U(x,y) = \ln x + y$$

- Setting up the Lagrangian

$$\mathbf{L} = \ln x + y + \lambda(I - p_x x - p_y y)$$

Asymmetry of the Gross Definitions

gives us the following first-order conditions:

$$\partial \mathbf{L} / \partial x = 1/x - \lambda p_x = 0$$

$$\partial \mathbf{L} / \partial y = 1 - \lambda p_y = 0$$

$$\partial \mathbf{L} / \partial \lambda = I - p_x x - p_y y = 0$$

- Manipulating the first two equations, we get

$$p_x x = p_y$$

Asymmetry of the Gross Definitions

- Inserting this into the budget constraint, we can find the Marshallian demand for *y*

$$p_y y = I - p_y$$

  - an increase in p_y causes a decline in spending on *y*
    - since p_x and I are unchanged, spending on *x* must rise ($\Rightarrow$ *x* and *y* are gross substitutes)
    - but spending on *y* is independent of p_x ($\Rightarrow$ *x* and *y* are independent of one another)

Net （净） Substitutes and Complements

- The concepts of net substitutes and complements focuses solely on substitution effects
  – two goods are net substitutes if

$$\left.\frac{\partial x_i}{\partial p_j}\right|_{U=\text{constant}} > 0$$

  – two goods are net complements if

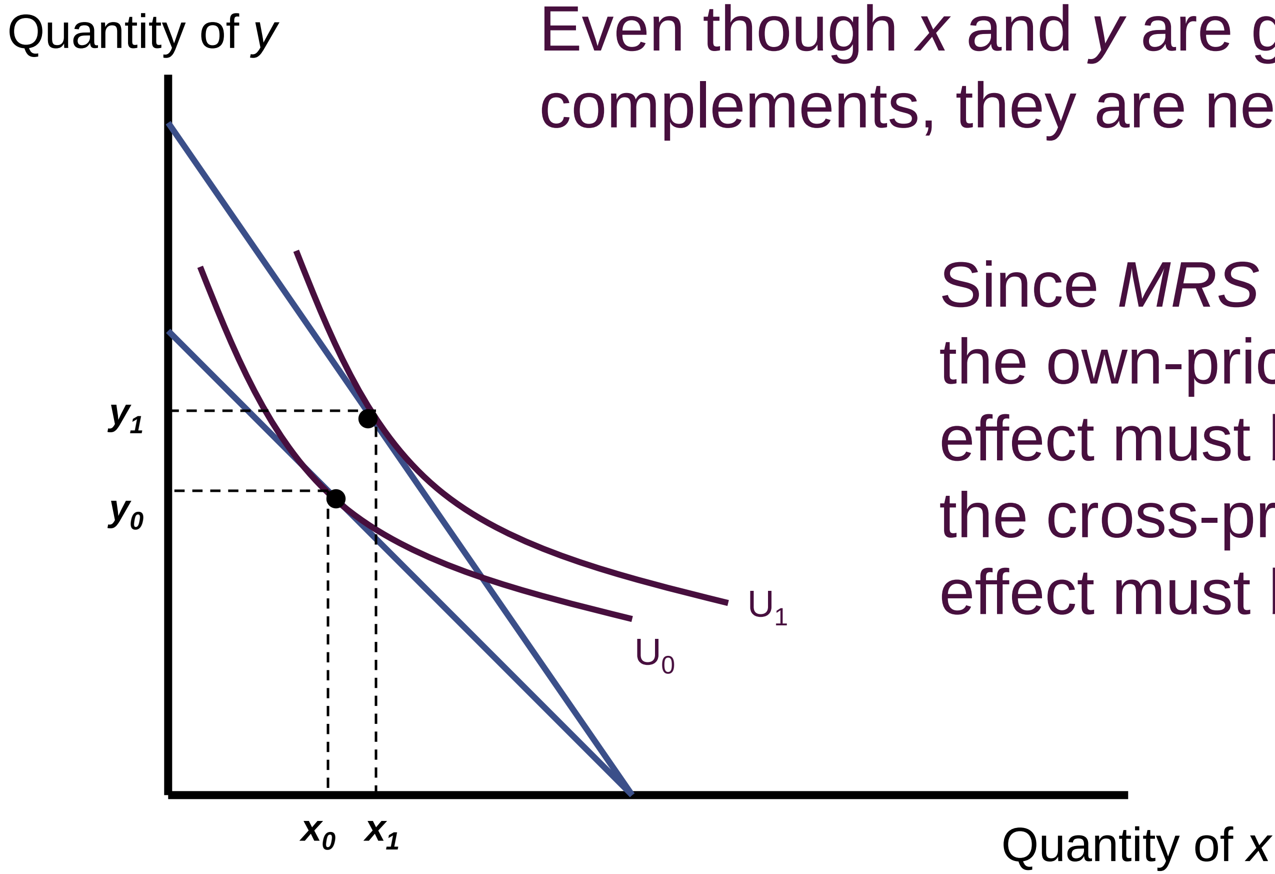
$$\left.\frac{\partial x_i}{\partial p_j}\right|_{U=\text{constant}} < 0$$

Net Substitutes and Complements

- This definition looks only at the shape of the indifference curve
- This definition is unambiguous because the definitions are perfectly symmetric

$$\left.\frac{\partial x_i}{\partial p_j}\right|_{U=\text{constant}} = \left.\frac{\partial x_j}{\partial p_i}\right|_{U=\text{constant}}$$

Gross Complements



Even though *x* and *y* are gross complements, they are net substitutes

Since *MRS* is diminishing, the own-price substitution effect must be negative so the cross-price substitution effect must be positive

Substitutability with Many Goods

- Once the utility-maximizing model is extended to may goods, a wide variety of demand patterns become possible
- According to Hicks' second law of demand, "most" goods must be substitutes

Substitutability with Many Goods

- To prove this, we can start with the compensated demand function

$$x^c(p_1,\ldots p_n,V)$$

- Applying Euler's theorem yields

$$p_1 \cdot \frac{\partial x_i^c}{\partial p_1} + p_2 \cdot \frac{\partial x_i^c}{\partial p_2} + \ldots + p_n \frac{\partial x_i^c}{\partial p_n} \equiv 0$$

Substitutability with Many Goods

- In elasticity terms, we get

$$e_{i1}^c + e_{i2}^c + \ldots + e_{in}^c \equiv 0$$

- Since the negativity of the substitution effect implies that $e_{ii}^c \leq 0$, it must be the case that

$$\sum_{j \neq i} e_{ij}^c \geq 0$$

Composite Commodities （skipped）

- In the most general case, an individual who consumes n goods will have demand functions that reflect $n(n+1)/2$ different substitution effects
- It is often convenient to group goods into larger aggregates
  - examples: food, clothing, “all other goods”

Composite Commodity Theorem

- Suppose that consumers choose among *n* goods
- The demand for x_1 will depend on the prices of the other *n*-1 commodities
- If all of these prices move together, it may make sense to lump them into a single composite commodity (*y*)

Composite Commodity Theorem

- Let $p_2^0 \ldots p_n^0$ represent the initial prices of these other commodities
  - assume that they all vary together (so that the relative prices of $x_2 \ldots x_n$ do not change)
- Define the composite commodity y to be total expenditures on $x_2 \ldots x_n$ at the initial prices

$$y = p_2^0 x_2 + p_3^0 x_3 + \ldots + p_n^0 x_n$$

Composite Commodity Theorem

- The individual's budget constraint is

$$I = p_1x_1 + p_2^0x_2 + \ldots + p_n^0x_n = p_1x_1 + y$$

- If we assume that all of the prices $p_2^0 \ldots p_n^0$ change by the same factor ($t > 0$) then the budget constraint becomes

$$I = p_1x_1 + tp_2^0x_2 + \ldots + tp_n^0x_n = p_1x_1 + ty$$

  – changes in p_1 or t induce substitution effects

Composite Commodity Theorem

- As long as $p_2^0 \ldots p_n^0$ move together, we can confine our examination of demand to choices between buying x_1 and "everything else"
- The theorem makes no prediction about how choices of $x_2 \ldots x_n$ behave
  - only focuses on total spending on $x_2 \ldots x_n$

Composite Commodity

- A composite commodity is a group of goods for which all prices move together
- These goods can be treated as a single commodity
  - the individual behaves as if he is choosing between other goods and spending on this entire composite group

Example: Composite Commodity

- Suppose that an individual receives utility from three goods:
  - food (x)
  - housing services (y), measured in hundreds of square feet
  - household operations (z), measured by electricity use
- Assume a CES utility function

Example: Composite Commodity

$$\text{utility} = U(x,y,z) = -\frac{1}{x} - \frac{1}{y} - \frac{1}{z}$$

- The Lagrangian technique can be used to derive demand functions

$$x = \frac{I}{p_x + \sqrt{p_x p_y} + \sqrt{p_x p_z}} \qquad y = \frac{I}{p_y + \sqrt{p_y p_x} + \sqrt{p_y p_z}}$$

$$z = \frac{I}{p_z + \sqrt{p_z p_x} + \sqrt{p_z p_y}}$$

Example: Composite Commodity

- If initially $I = 100$, $p_x = 1$, $p_y = 4$, and $p_z = 1$, then
- $x^* = 25$, $y^* = 12.5$, $z^* = 25$
  - $25 is spent on food and $75 is spent on housing-related needs

Example: Composite Commodity

- If we assume that the prices of housing services (p_y) and electricity (p_z) move together, we can use their initial prices to define the "composite commodity" housing (h)

$$h = 4y + 1z$$

- The initial quantity of housing is the total spent on housing (75)

Example: Composite Commodity

- Now x can be shown as a function of I, p_x, and p_h

$$x = \frac{I}{p_y + 3\sqrt{p_x p_h}}$$

- If $I = 100$, $p_x = 1$, $p_y = 4$, and $p_h = 1$, then $x^* = 25$ and spending on housing (h^*) = 75

Example: Composite Commodity

- If p_y rises to 16 and p_z rises to 4 (with p_x remaining at 1), p_h would also rise to 4
- The demand for *x* would fall to

$$x^* = \frac{100}{1+3\sqrt{4}} = \frac{100}{7}$$

- Housing purchases would be given by

$$P_h h^* = 100 - \frac{100}{7} = \frac{600}{7}$$

Example: Composite Commodity

- Since $p_h = 4$, $h^* = 150/7$
- If $I = 100$, $p_x = 1$, $p_y = 16$, and $p_z = 4$, the individual demand functions show that

$$x^* = 100/7,\ y^* = 100/28,\ z^* = 100/14$$

- This means that the amount of h that is consumed can also be computed as

$$h^* = 4y^* + 1z^* = 150/7$$

Household Production Model

- Assume that individuals do not receive utility directly from the goods they purchase in the market
- Utility is received when the individual produces goods by combining market goods with time inputs
  – raw beef and uncooked potatoes yield no utility until they are cooked together to produce stew

Household Production Model

- Assume that there are three goods that a person might want to purchase in the market: *x*, *y*, and *z*
  - these goods provide no direct utility
  - these goods can be combined by the individual to produce either of two home-produced goods: a_1 or a_2
    - the technology of this household production can be represented by a production function

Household Production Model

- The individual's goal is to choose x,y, and z so as to maximize utility

$$\text{utility} = U(a_1,a_2)$$

subject to the production functions

$$a_1 = f_1(x,y,z)$$
$$a_2 = f_2(x,y,z)$$

and a financial budget constraint

$$p_x x + p_y y + p_z z = I$$

Household Production Model（skipped）

- Two important insights from this general model can be drawn
  - because the production functions are measurable, households can be treated as “multi-product” firms
  - because consuming more a_1 requires more use of x, y, and z, this activity has an opportunity cost in terms of the amount of a_2 that can be produced

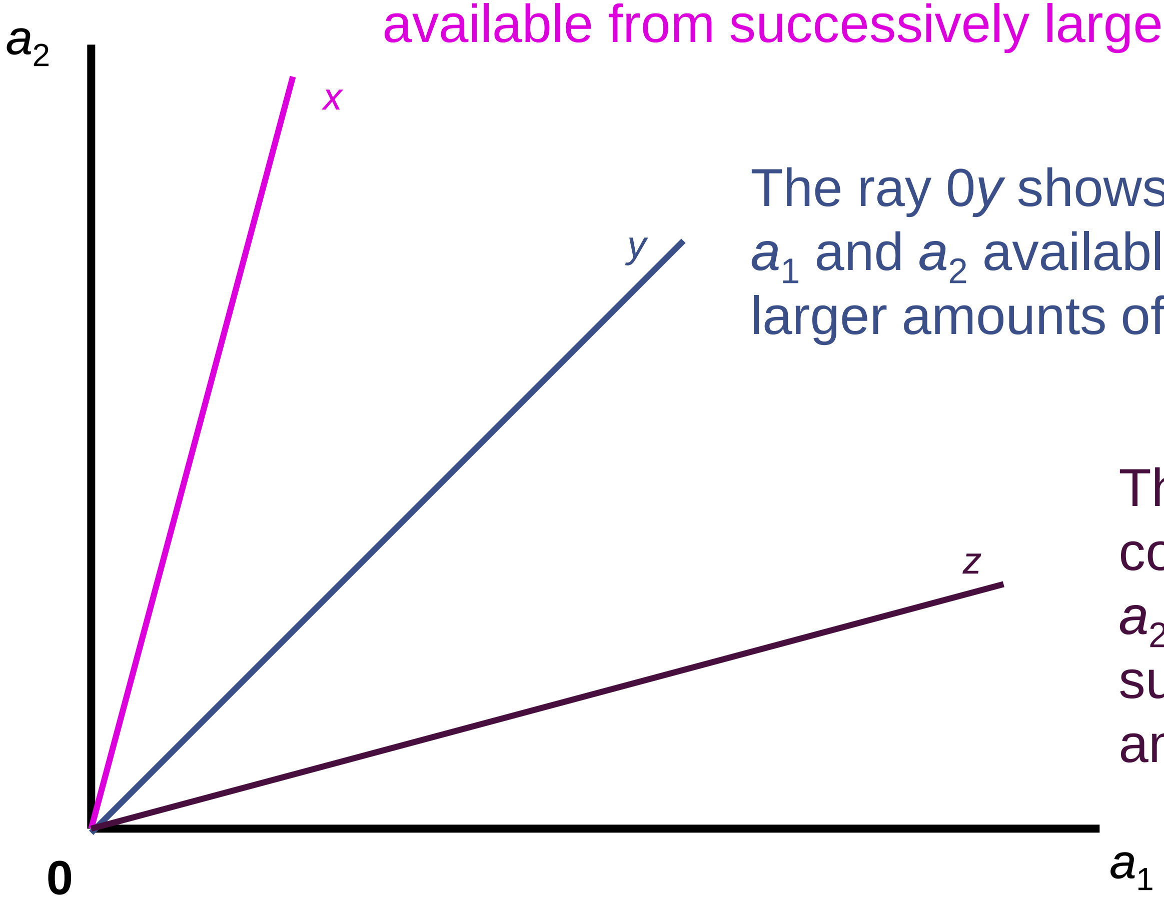
The Linear Attributes Model

- In this model, it is the attributes of goods that provide utility to individuals
- Each good has a fixed set of attributes
- The model assumes that the production equations for a_1 and a_2 have the form

$$a_1 = a_x^1 x + a_y^1 y + a_z^1 z$$

$$a_2 = a_x^2 x + a_y^2 y + a_z^2 z$$

The Linear Attributes Model



The ray 0*x* shows the combinations of a_1 and a_2 available from successively larger amounts of good *x*

The ray 0*y* shows the combinations of a_1 and a_2 available from successively larger amounts of good *y*

The ray 0*z* shows the combinations of a_1 and a_2 available from successively larger amounts of good *z*

The Linear Attributes Model

- If the individual spends all of his or her income on good *x*

$$x^* = I/p_x$$

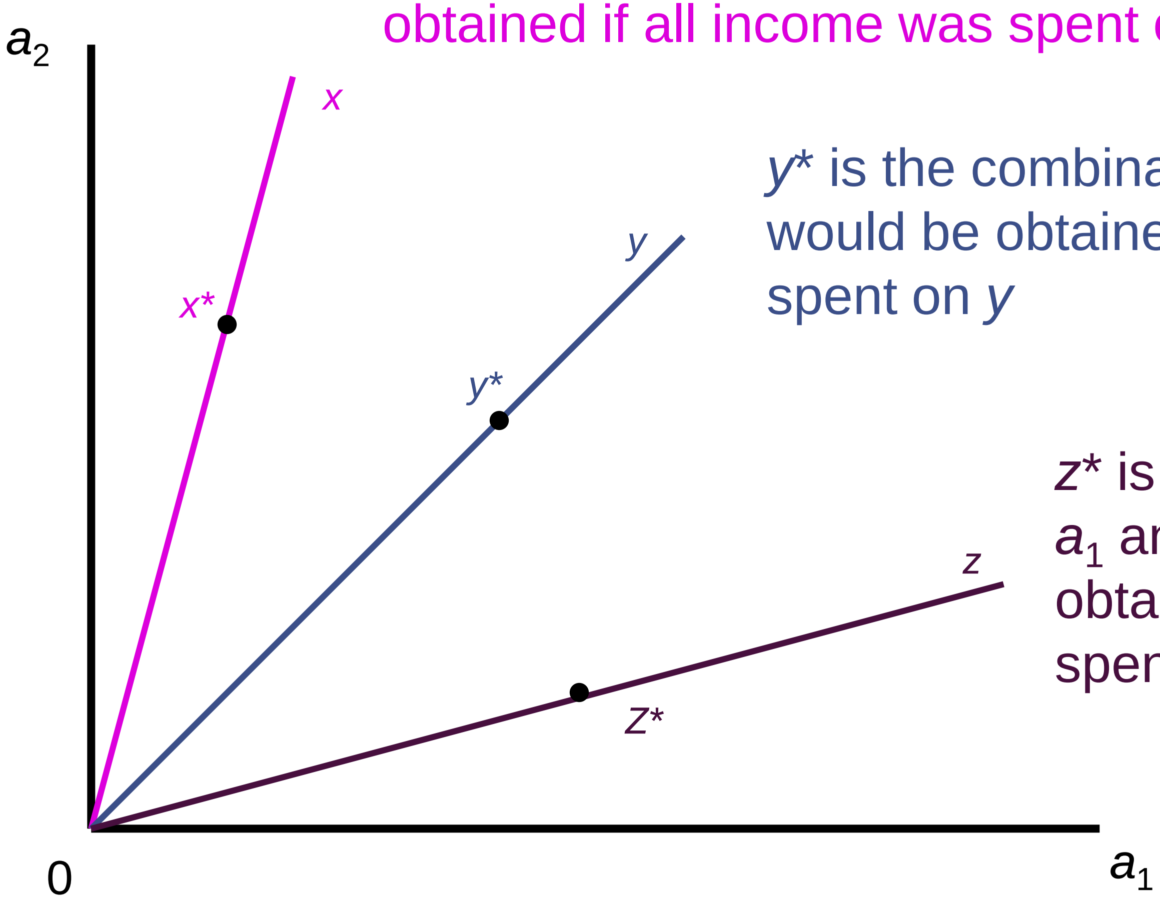
- That will yield

$$a_1^* = a_x^1 x^* = (a_x^1 I)/p_x$$

$$a_2^* = a_x^2 x^* = (a_x^2 I)/p_x$$

The Linear Attributes Model

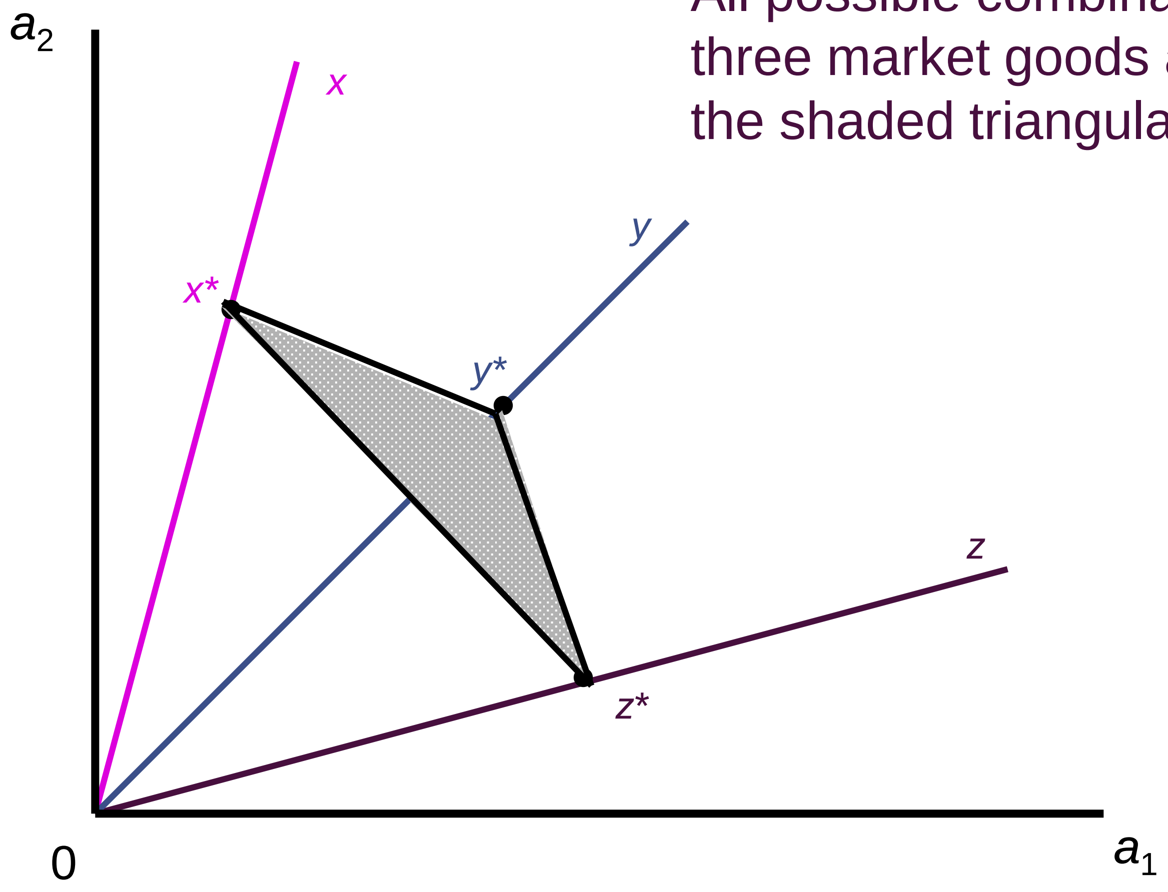
x^* is the combination of a_1 and a_2 that would be obtained if all income was spent on x



y^* is the combination of a_1 and a_2 that would be obtained if all income was spent on y

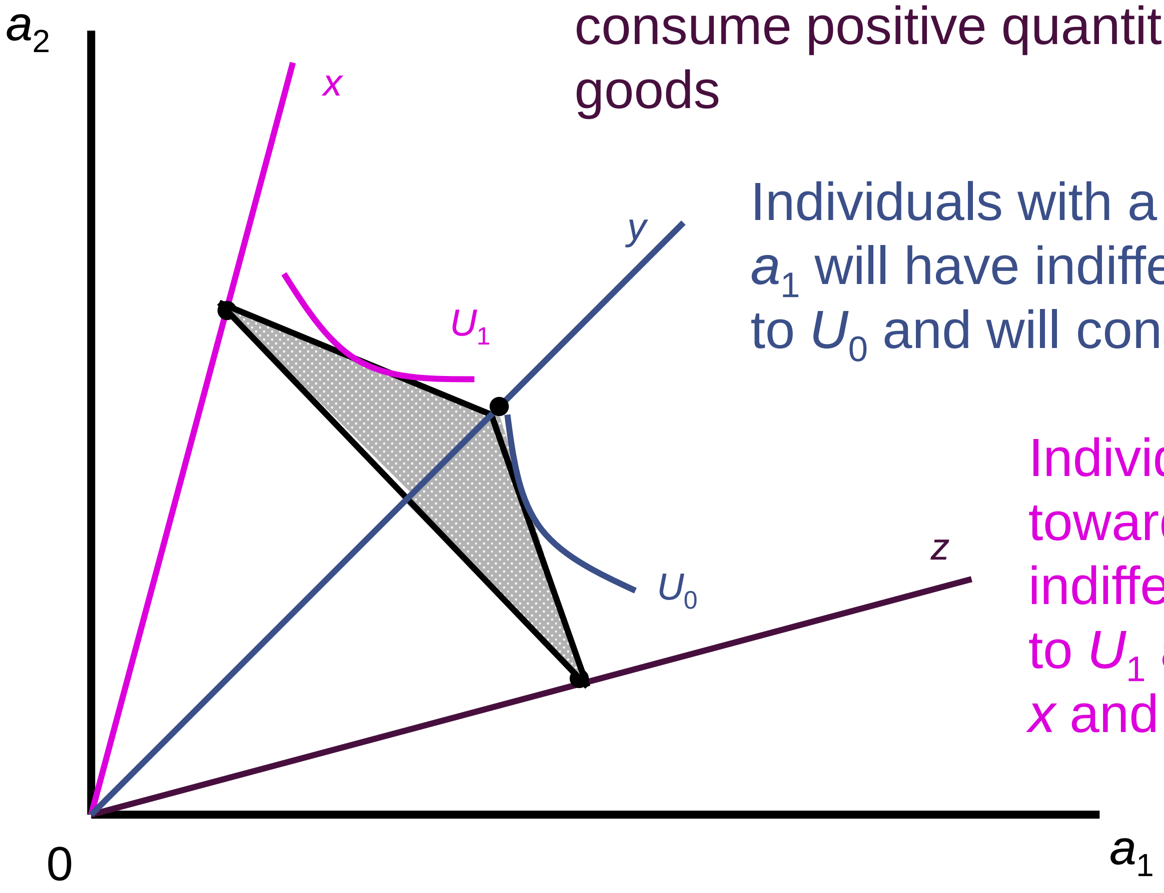
z^* is the combination of a_1 and a_2 that would be obtained if all income was spent on z

The Linear Attributes Model



All possible combinations from mixing the three market goods are represented by the shaded triangular area *x*y*z**

The Linear Attributes Model



A utility-maximizing individual would never consume positive quantities of all three goods

Individuals with a preference toward a_1 will have indifference curves similar to U_0 and will consume only *y* and *z*

Individuals with a preference toward a_0 will have indifference curves similar to U_1 and will consume only *x* and *y*

The Linear Attributes Model

- The model predicts that corner solutions (where individuals consume zero amounts of some commodities) will be relatively common
  - especially in cases where individuals attach value to fewer attributes than there are market goods to choose from
- Consumption patterns may change abruptly if income, prices, or preferences change

Important Points to Note:

- When there are only two goods, the income and substitution effects from the change in the price of one good (p_y) on the demand for another good (x) usually work in opposite directions
  - the sign of $\partial x/\partial p_y$ is ambiguous
    - the substitution effect is positive
    - the income effect is negative

Important Points to Note:

- In cases of more than two goods, demand relationships can be specified in two ways
  - two goods are gross substitutes if $\partial x_i / \partial p_j > 0$ and gross complements if $\partial x_i / \partial p_j < 0$
  - because these price effects include income effects, they may not be symmetric
    - it is possible that $\partial x_i / \partial p_j \neq \partial x_j / \partial p_i$

Important Points to Note:

- Focusing only on the substitution effects from price changes does provide a symmetric definition
  - two goods are net substitutes if $\partial x_i^c/\partial p_j > 0$ and net complements if $\partial x_i^c/\partial p_j < 0$
  - because $\partial x_i^c/\partial p_j = \partial x_j^c/\partial p_i$, there is no ambiguity
  - Hicks' second law of demand shows that net substitutes are more prevalent

Important Points to Note:

- If a group of goods has prices that always move in unison, expenditures on these goods can be treated as a "composite commodity" whose "price" is given by the size of the proportional change in the composite goods' prices

Important Points to Note:

- An alternative way to develop the theory of choice among market goods is to focus on the ways in which market goods are used in household production to yield utility-providing attributes
  - this may provide additional insights into relationships among goods