Chapter 12

GENERAL EQUILIBRIUM AND WELFARE

Perfectly Competitive Price System

- We will assume that all markets are perfectly competitive
  - there is some large number of homogeneous goods in the economy
    - both consumption goods and factors of production
  - each good has an equilibrium price
  - there are no transaction or transportation costs
  - individuals and firms have perfect information

Law of One Price

- A homogeneous good trades at the same price no matter who buys it or who sells it
  - if one good traded at two different prices, demanders would rush to buy the good where it was cheaper and firms would try to sell their output where the price was higher
    - these actions would tend to equalize the price of the good

Assumptions of Perfect Competition

- There are a large number of people buying any one good
  - each person takes all prices as given and seeks to maximize utility given his budget constraint
- There are a large number of firms producing each good
  - each firm takes all prices as given and attempts to maximize profits

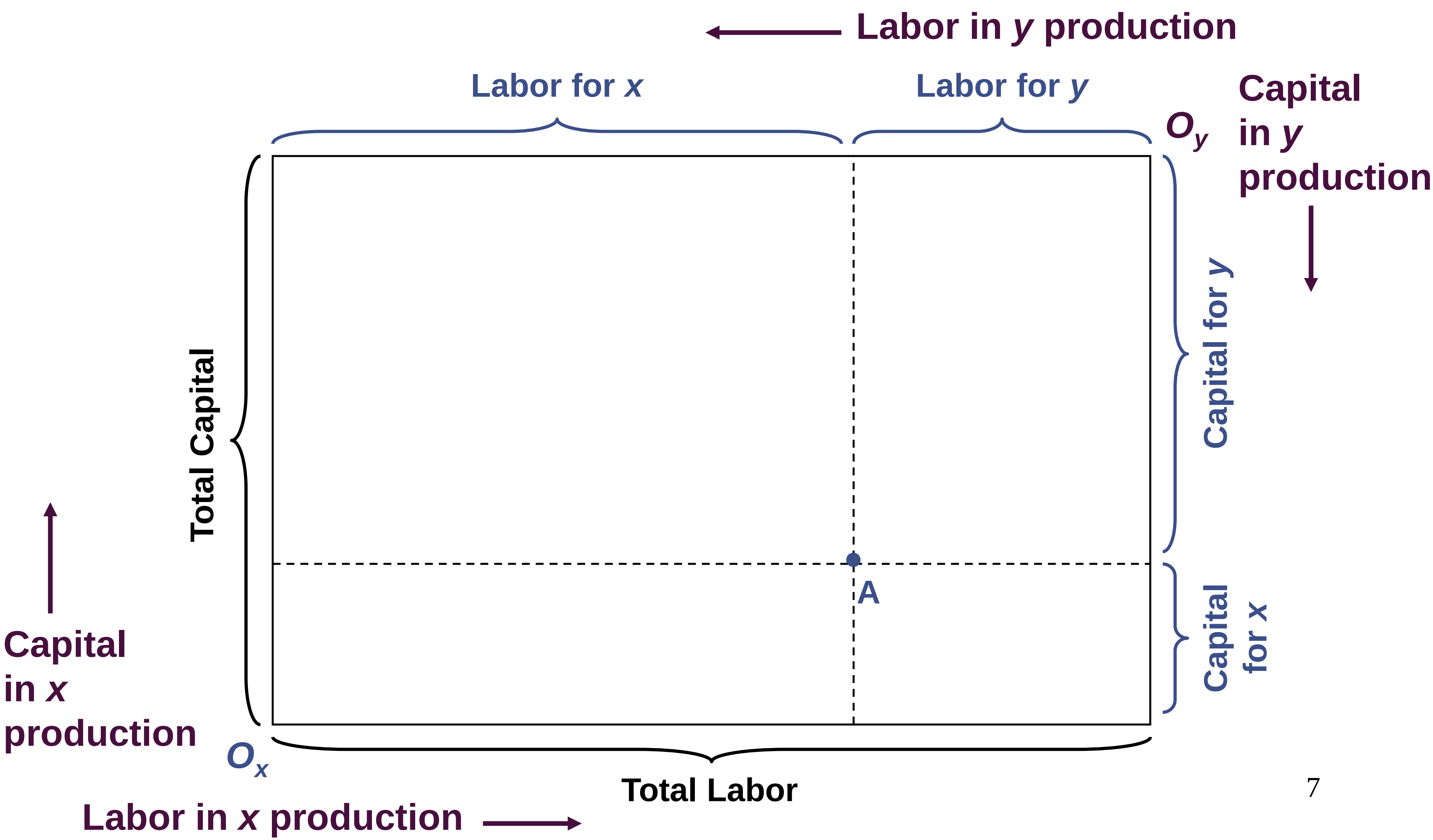
General Equilibrium

- Assume that there are only two goods, *x* and *y*
- All individuals are assumed to have identical preferences
  - represented by an indifference map
- The production possibility curve can be used to show how outputs and inputs are related

Edgeworth Box Diagram

- Construction of the production possibility curve for *x* and *y* starts with the assumption that the amounts of *k* and *l* are fixed
- An Edgeworth box shows every possible way the existing *k* and *l* might be used to produce *x* and *y*
  - any point in the box represents a fully employed allocation of the available resources to *x* and *y*

Edgeworth Box Diagram



Edgeworth Box Diagram

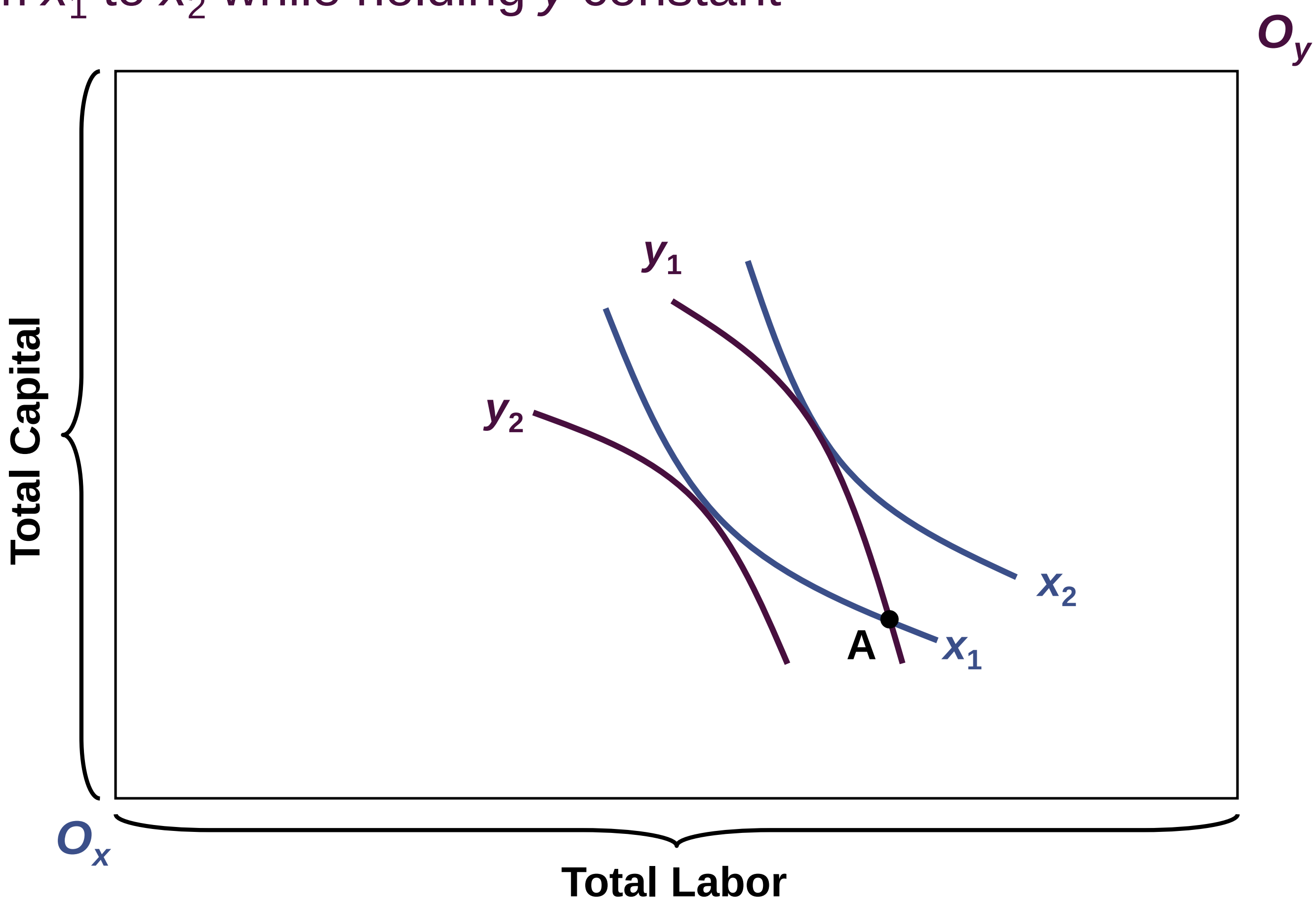
- Many of the allocations in the Edgeworth box are technically inefficient
  - it is possible to produce more *x* and more *y* by shifting capital and labor around
- We will assume that competitive markets will not exhibit inefficient input choices
- We want to find the efficient allocations
  - they illustrate the actual production outcomes

Edgeworth Box Diagram

- We will use isoquant maps for the two goods
  - the isoquant map for good x uses O_x as the origin
  - the isoquant map for good y uses O_y as the origin
- The efficient allocations will occur where the isoquants are tangent to one another

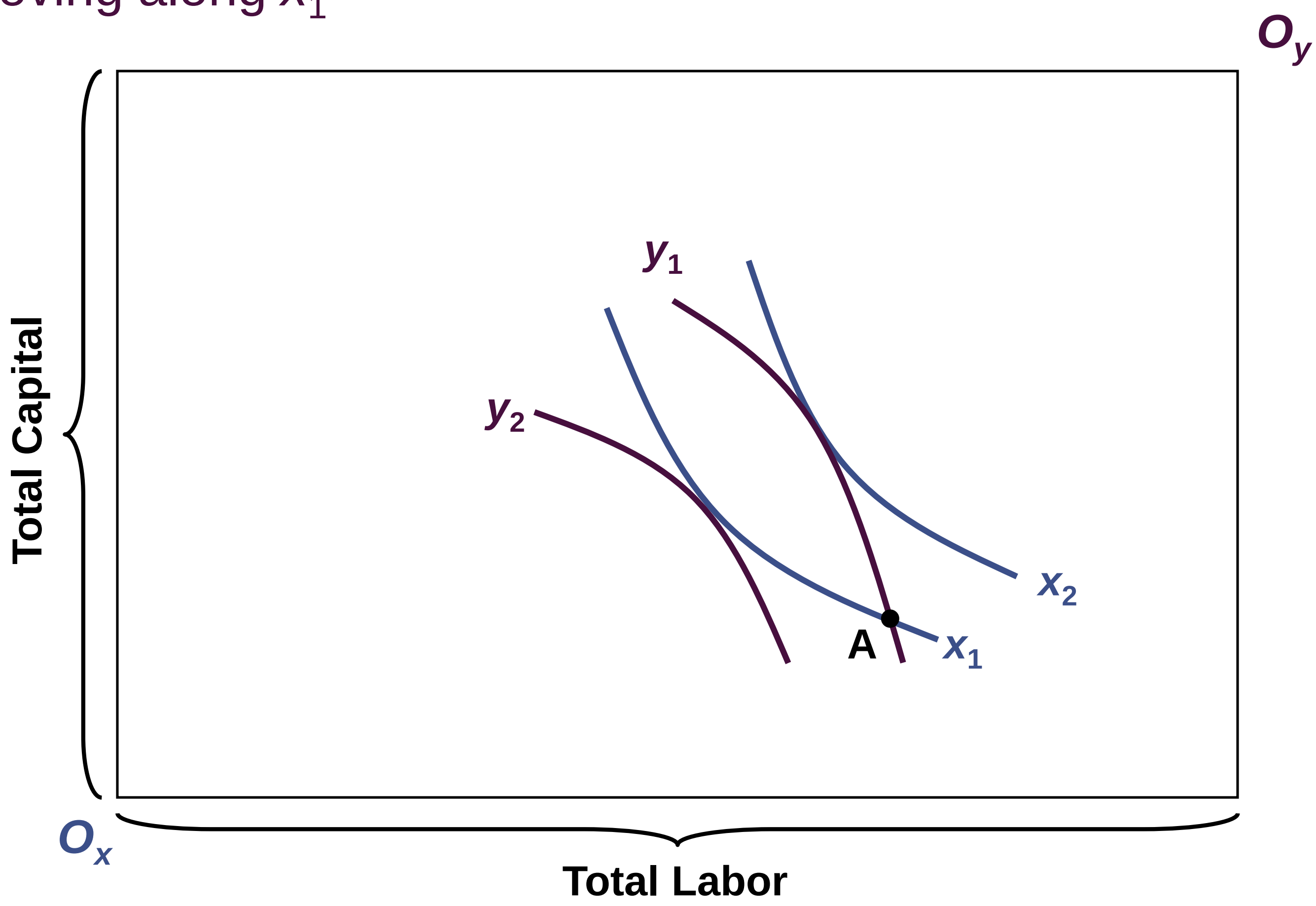
Edgeworth Box Diagram

Point *A* is inefficient because, by moving along y_1, we can increase x from x_1 to x_2 while holding y constant



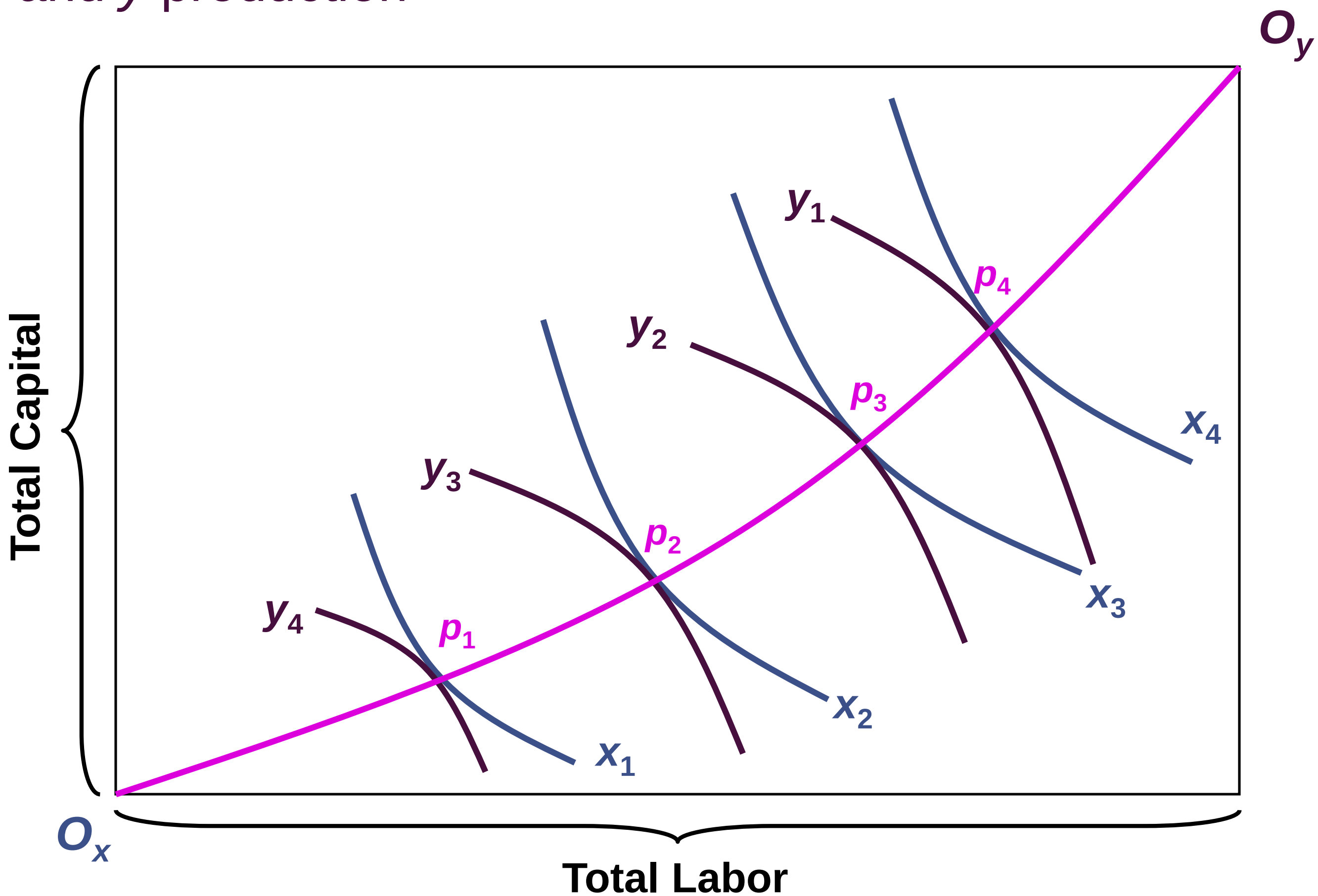
Edgeworth Box Diagram

We could also increase y from y_1 to y_2 while holding x constant by moving along x_1



Edgeworth Box Diagram

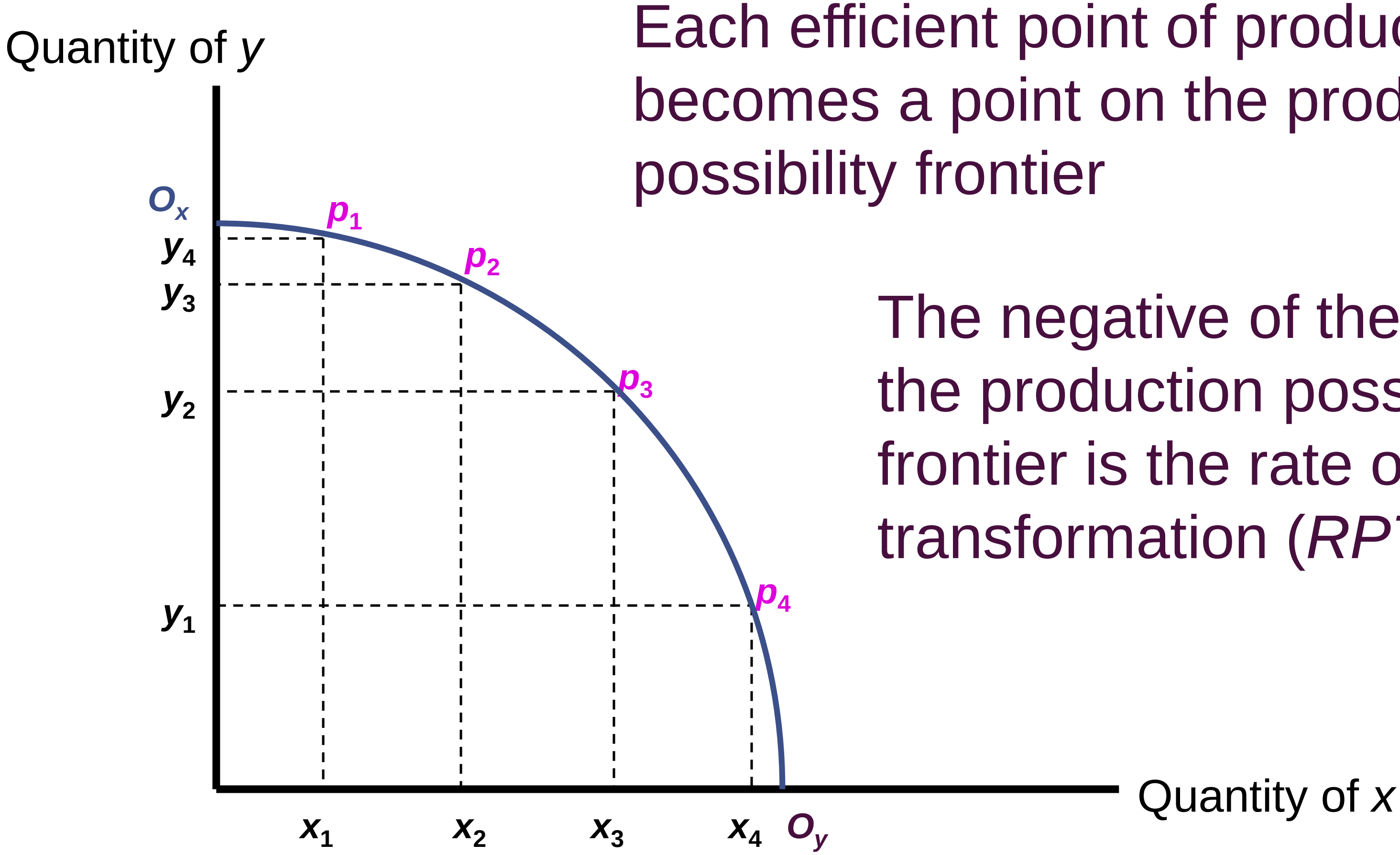
At each efficient point, the *RTS* (of *k* for *l*) is equal in both *x* and *y* production



Production Possibility Frontier

- The locus of efficient points shows the maximum output of *y* that can be produced for any level of *x*
  - we can use this information to construct a production possibility frontier
    - shows the alternative outputs of *x* and *y* that can be produced with the fixed capital and labor inputs that are employed efficiently

Production Possibility Frontier



Each efficient point of production becomes a point on the production possibility frontier

The negative of the slope of the production possibility frontier is the rate of product transformation (*RPT*)

Rate of Product Transformation

- The rate of product transformation (*RPT*) between two outputs is the negative of the slope of the production possibility frontier

$$RPT \text{ (of } x \text{ for } y) = -\text{ slope of production possibility frontier}$$

$$RPT \text{ (of } x \text{ for } y) = -\frac{dy}{dx} \text{ (along } O_xO_y)$$

Rate of Product Transformation

- The rate of product transformation shows how *x* can be technically traded for *y* while continuing to keep the available productive inputs efficiently employed

Shape of the Production Possibility Frontier

- The production possibility frontier shown earlier exhibited an increasing *RPT*
  - this concave shape will characterize most production situations
- *RPT* is equal to the ratio of MC_x to MC_y

Shape of the Production Possibility Frontier

- Suppose that the costs of any output combination are $C(x,y)$
  - along the production possibility frontier, $C(x,y)$ is constant
- We can write the total differential of the cost function as

$$dC = \frac{\partial C}{\partial x} \cdot dx + \frac{\partial C}{\partial y} \cdot dy = 0$$

Shape of the Production Possibility Frontier

- Rewriting, we get

$$RPT = -\frac{dy}{dx}\,(\text{along } O_xO_y) = \frac{\partial C / \partial x}{\partial C / \partial y} = \frac{MC_x}{MC_y}$$

- The *RPT* is a measure of the relative marginal costs of the two goods

Shape of the Production Possibility Frontier

- As production of *x* rises and production of *y* falls, the ratio of MC_x to MC_y rises
  - this occurs if both goods are produced under diminishing returns
    - increasing the production of *x* raises MC_x, while reducing the production of *y* lowers MC_y
  - this could also occur if some inputs were more suited for *x* production than for *y* production

Shape of the Production Possibility Frontier

- But we have assumed that inputs are homogeneous
- We need an explanation that allows homogeneous inputs and constant returns to scale
- The production possibility frontier will be concave if goods *x* and *y* use inputs in different proportions

Opportunity Cost

- The production possibility frontier demonstrates that there are many possible efficient combinations of two goods
- Producing more of one good necessitates lowering the production of the other good
  - this is what economists mean by opportunity cost

Opportunity Cost

- The opportunity cost of one more unit of *x* is the reduction in *y* that this entails
- Thus, the opportunity cost is best measured as the *RPT* (of *x* for *y*) at the prevailing point on the production possibility frontier
  - this opportunity cost rises as more *x* is produced

Concavity of the Production Possibility Frontier

- Suppose that the production of *x* and *y* depends only on labor and the production functions are

$$x = f(l_x) = l_x^{0.5} \qquad\qquad y = f(l_y) = l_y^{0.5}$$

- If labor supply is fixed at 100, then

$$l_x + l_y = 100$$

- The production possibility frontier is

$$x^2 + y^2 = 100 \qquad \text{for } x,y \geq 0$$

Concavity of the Production Possibility Frontier

- The *RPT* can be calculated by taking the total differential:

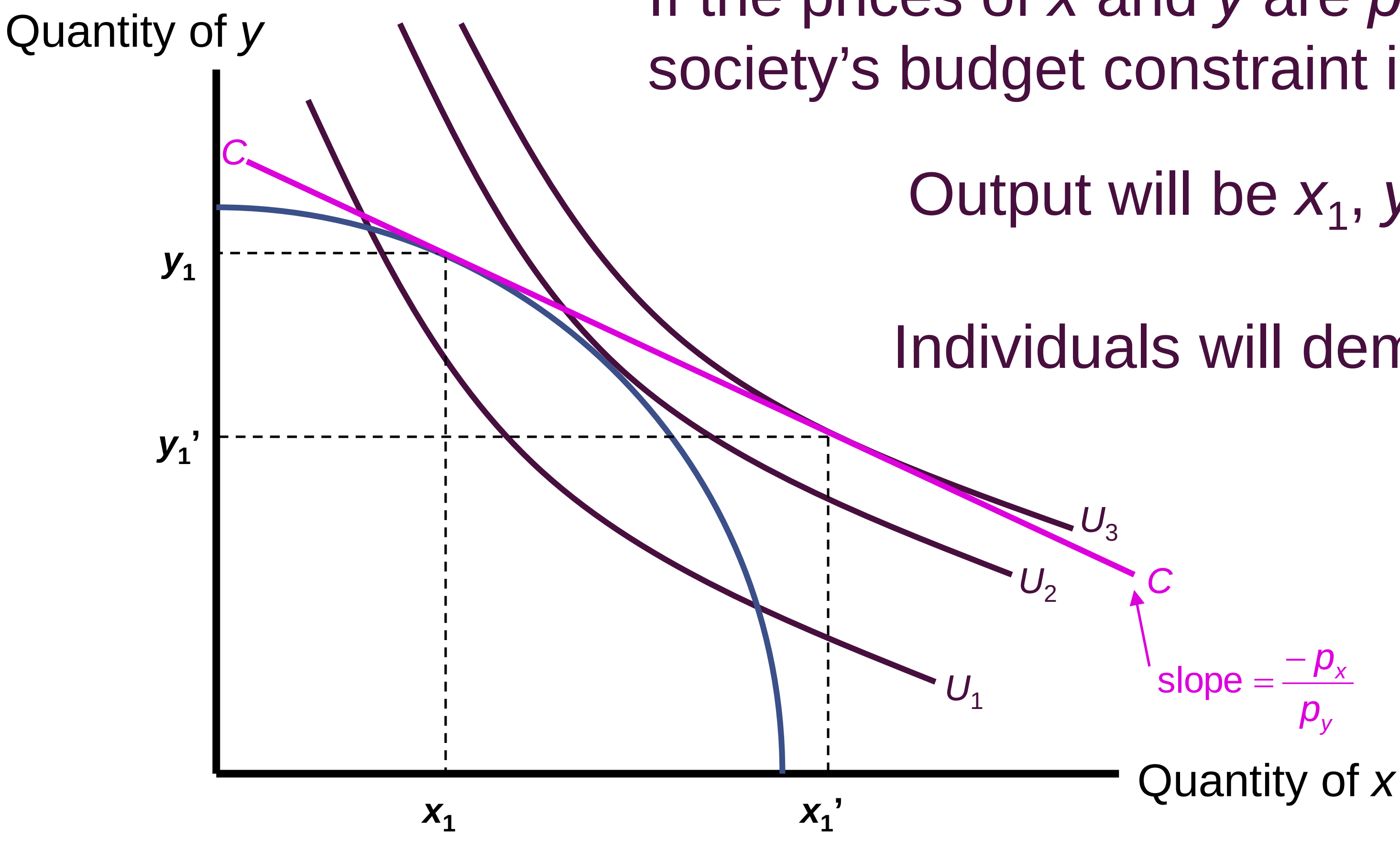
$$2xdx + 2ydy = 0 \quad \text{or} \quad RPT = \frac{-dy}{dx} = \frac{-(-2x)}{2y} = \frac{x}{y}$$

- The slope of the production possibility frontier increases as *x* output increases
  – the frontier is concave

Determination of Equilibrium Prices

- We can use the production possibility frontier along with a set of indifference curves to show how equilibrium prices are determined
  - the indifference curves represent individuals’ preferences for the two goods

Determination of Equilibrium Prices



If the prices of x and y are p_x and p_y, society's budget constraint is C

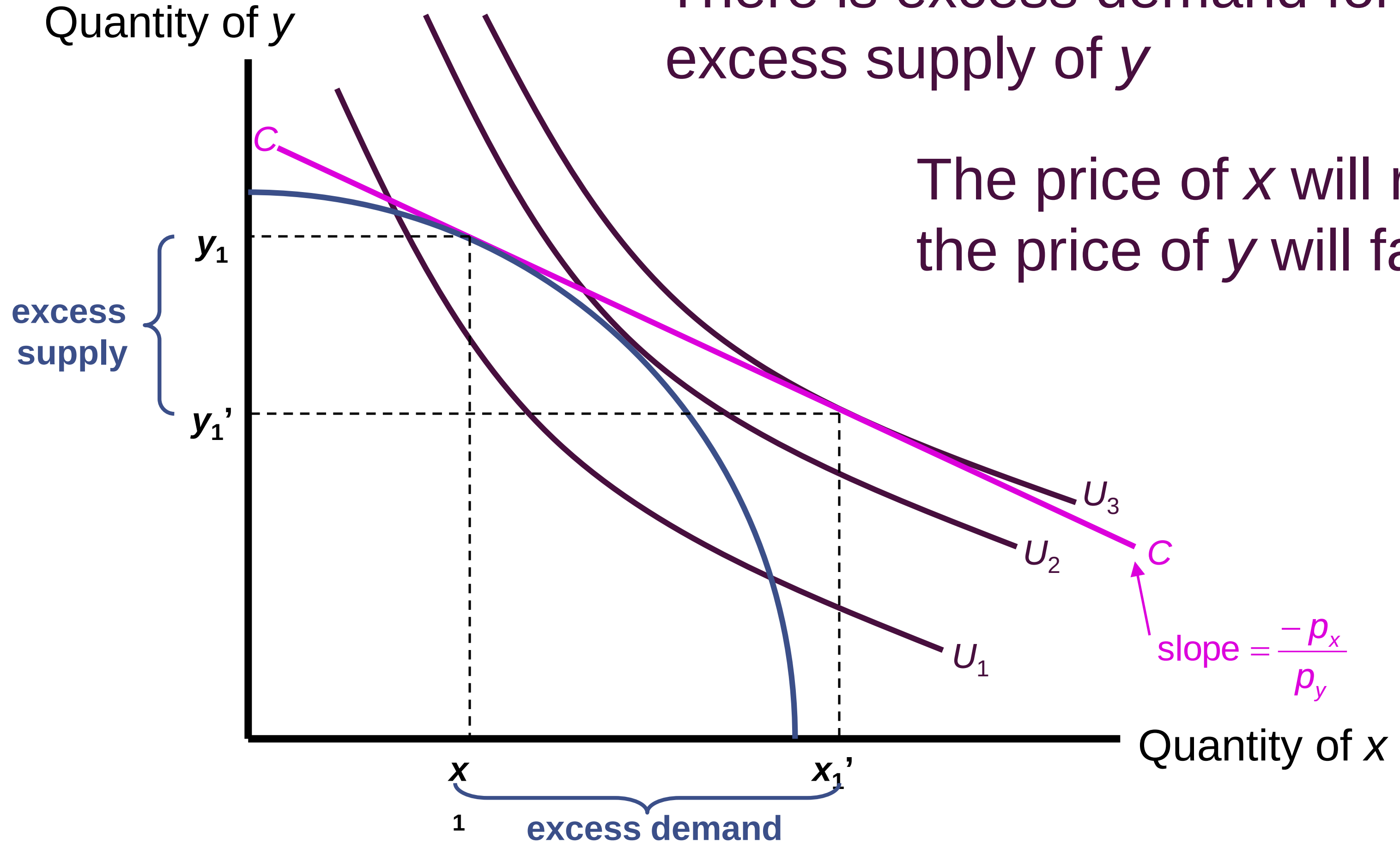
Output will be x_1, y_1

Individuals will demand x_1', y_1'

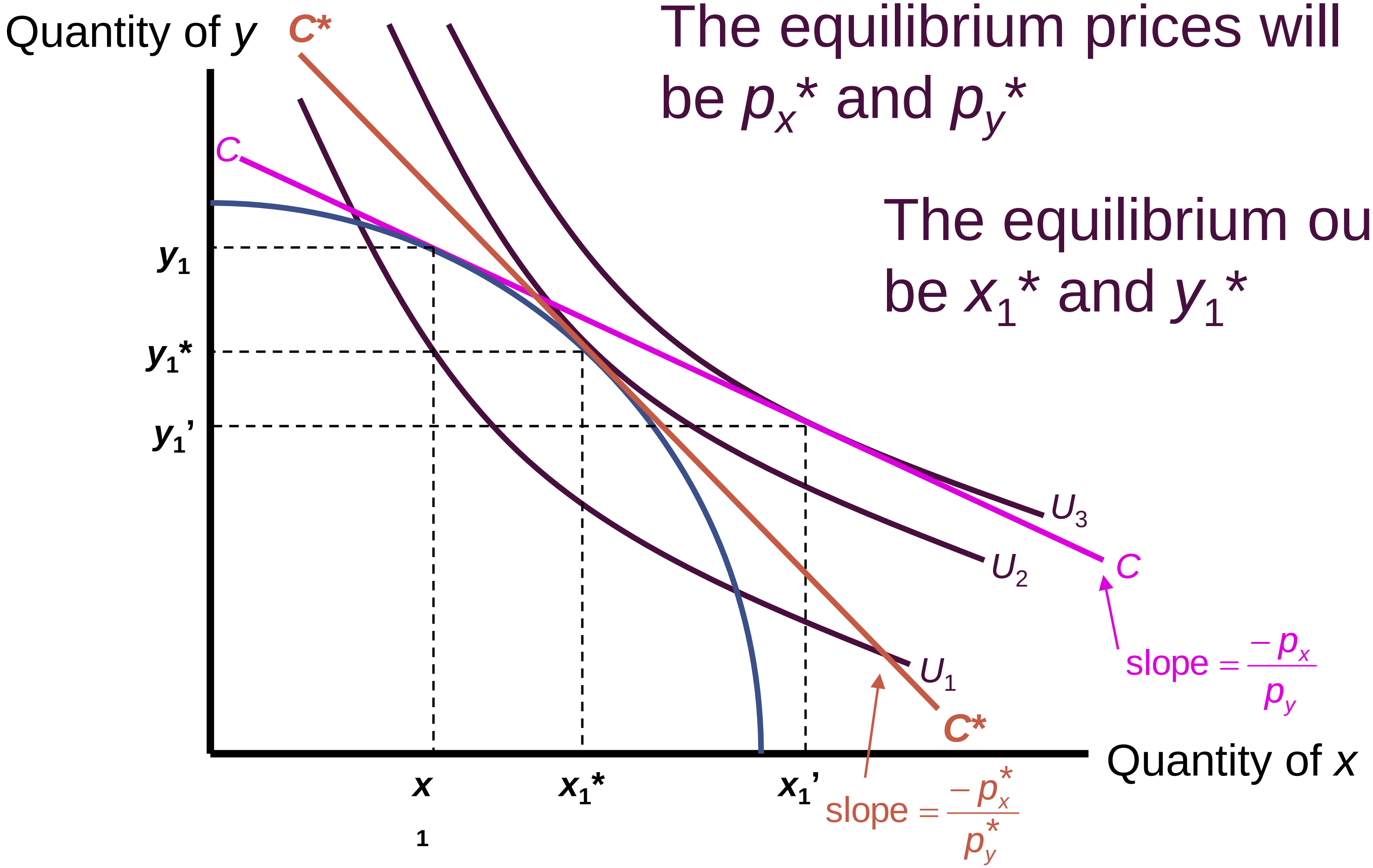
Determination of Equilibrium Prices

There is excess demand for *x* and excess supply of *y*

The price of *x* will rise and the price of *y* will fall



Determination of Equilibrium Prices



The equilibrium prices will be p_x* and p_y*

The equilibrium output will be x_1* and y_1*

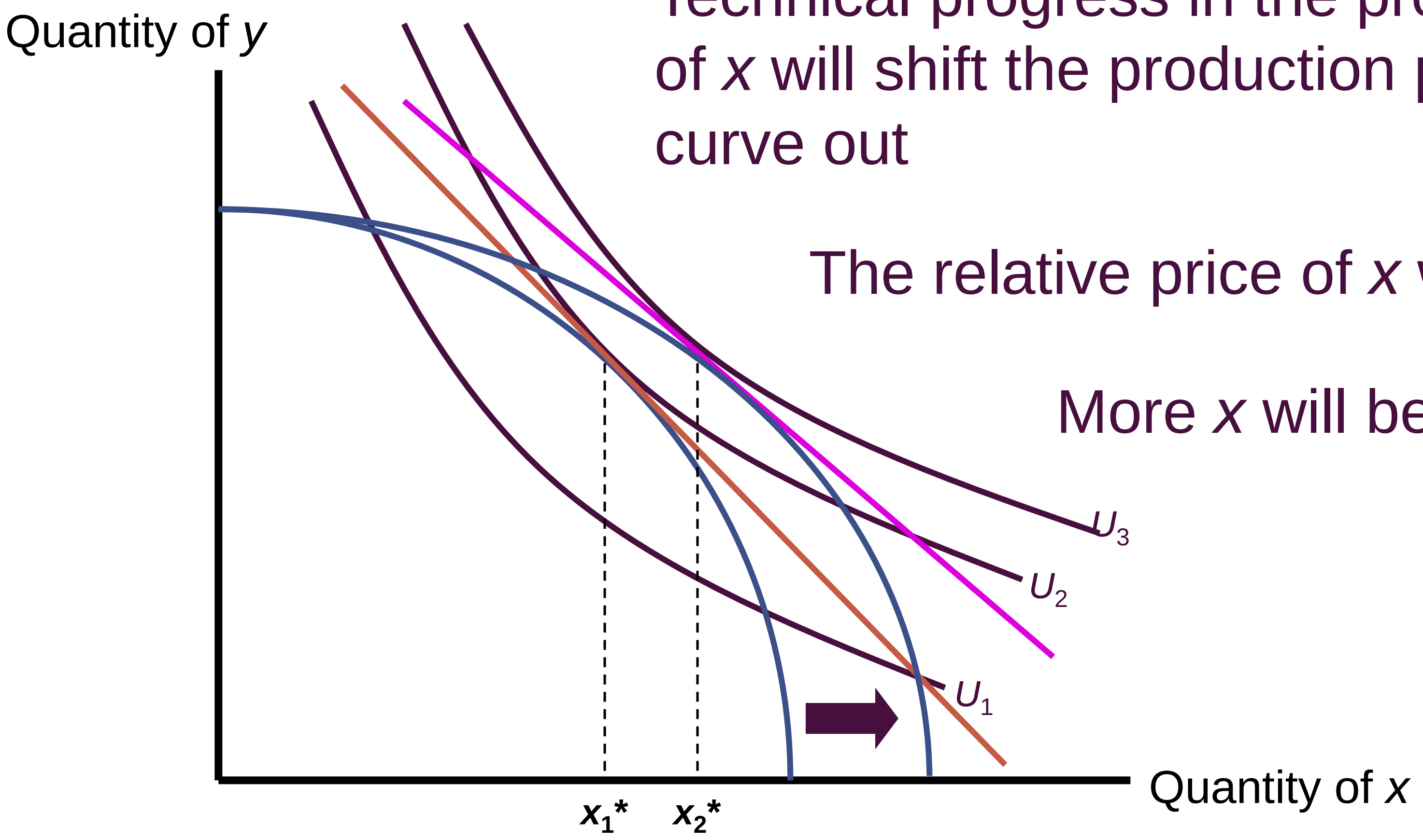
Comparative Statics Analysis

- The equilibrium price ratio will tend to persist until either preferences or production technologies change
- If preferences were to shift toward good x, p_x/p_y would rise and more x and less y would be produced
  – we would move in a clockwise direction along the production possibility frontier

Comparative Statics Analysis

- Technical progress in the production of good *x* will shift the production possibility curve outward
  - this will lower the relative price of *x*
  - more *x* will be consumed
    - if *x* is a normal good
  - the effect on *y* is ambiguous

Technical Progress in the Production of x



Technical progress in the production of *x* will shift the production possibility curve out

The relative price of *x* will fall

More *x* will be consumed

General Equilibrium Pricing

- Suppose that the production possibility frontier can be represented by

$$x^2 + y^2 = 100$$

- Suppose also that the community's preferences can be represented by

$$U(x,y) = x^{0.5}y^{0.5}$$

General Equilibrium Pricing

- Profit-maximizing firms will equate *RPT* and the ratio of p_x/p_y

$$RPT = \frac{x}{y} = \frac{p_x}{p_y}$$

- Utility maximization requires that

$$MRS = \frac{y}{x} = \frac{p_x}{p_y}$$

General Equilibrium Pricing

- Equilibrium requires that firms and individuals face the same price ratio

$$RPT = \frac{x}{y} = \frac{p_x}{p_y} = \frac{y}{x} = MRS$$

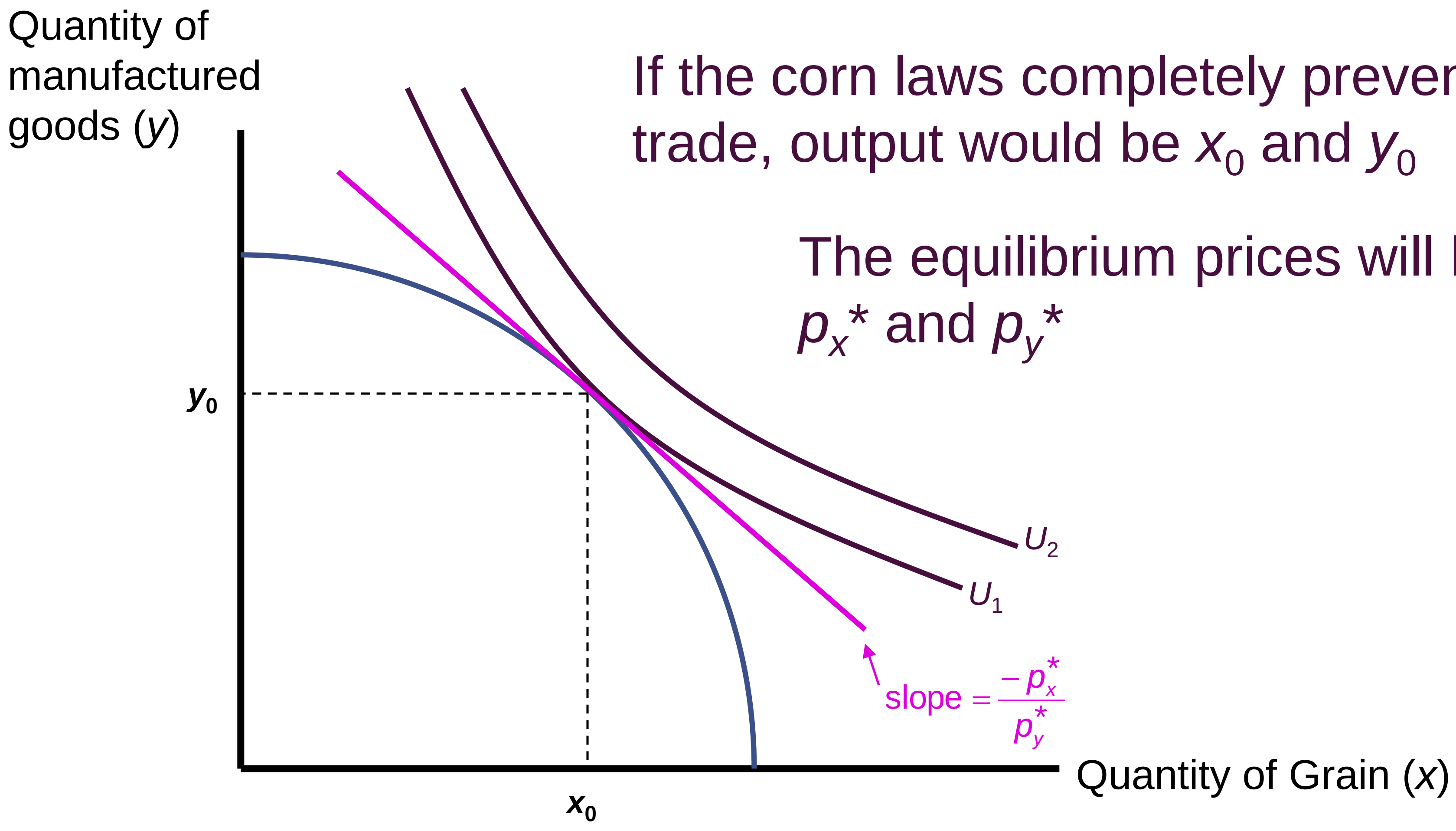
or

$$x^* = y^*$$

The Corn Laws Debate

- High tariffs on grain imports were imposed by the British government after the Napoleonic wars
- Economists debated the effects of these “corn laws” between 1829 and 1845
  - what effect would the elimination of these tariffs have on factor prices?

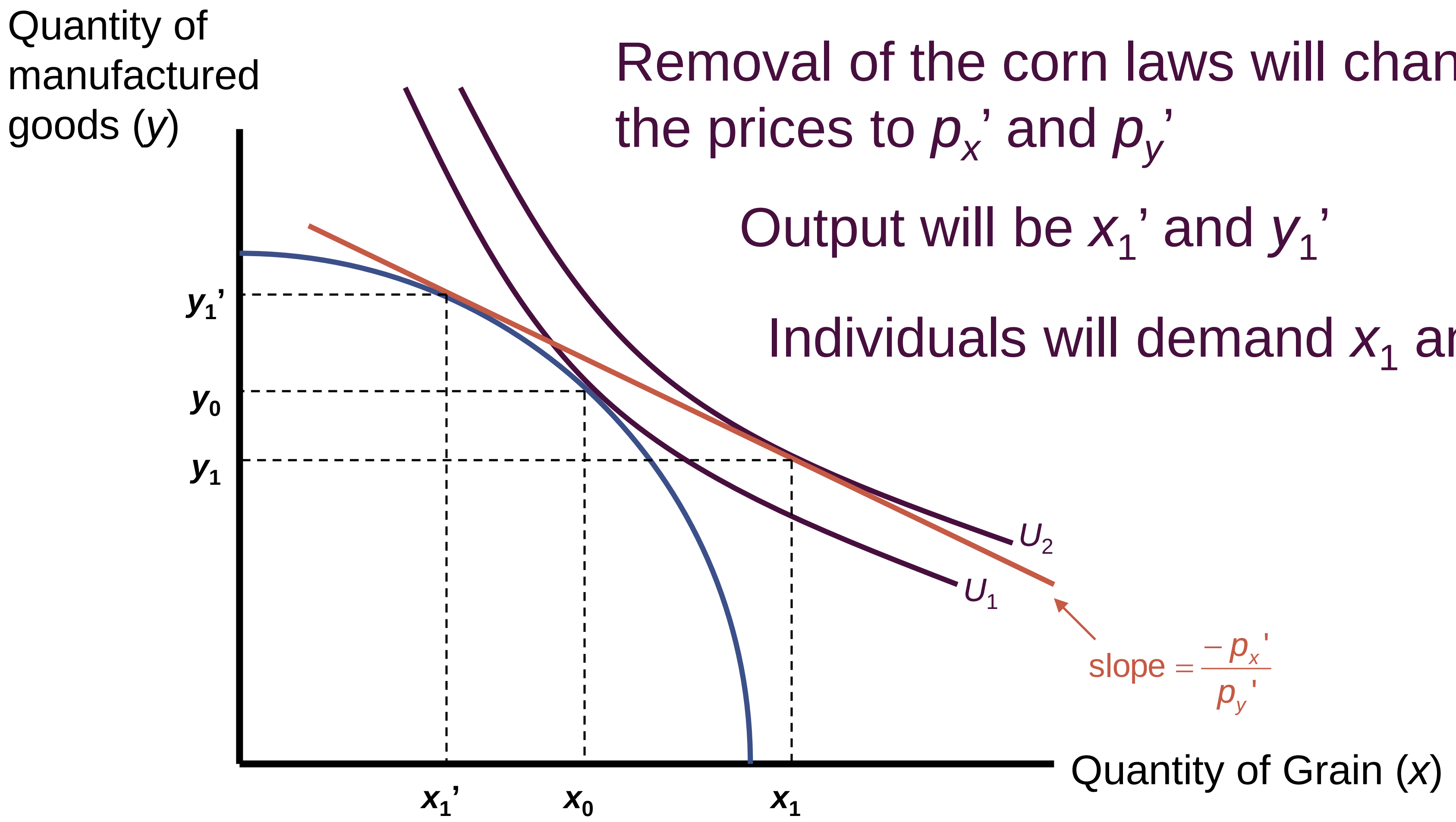
The Corn Laws Debate



If the corn laws completely prevented trade, output would be x_0 and y_0

The equilibrium prices will be p_x^* and p_y^*

The Corn Laws Debate



Removal of the corn laws will change the prices to p_x' and p_y'

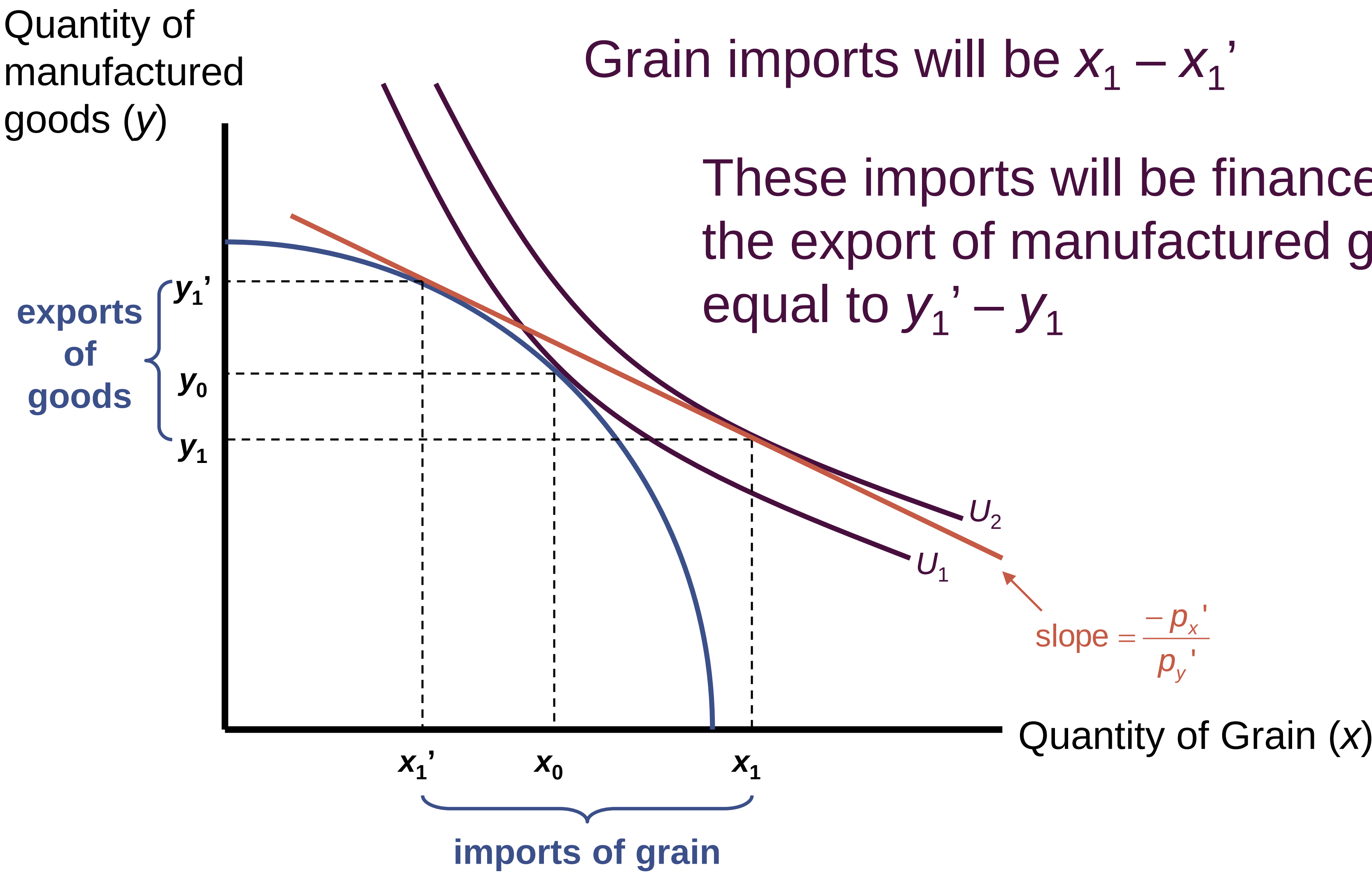
Output will be x_1' and y_1'

Individuals will demand x_1 and y_1

The Corn Laws Debate

Grain imports will be $x_1 - x_1'$

These imports will be financed by the export of manufactured goods equal to $y_1' - y_1$

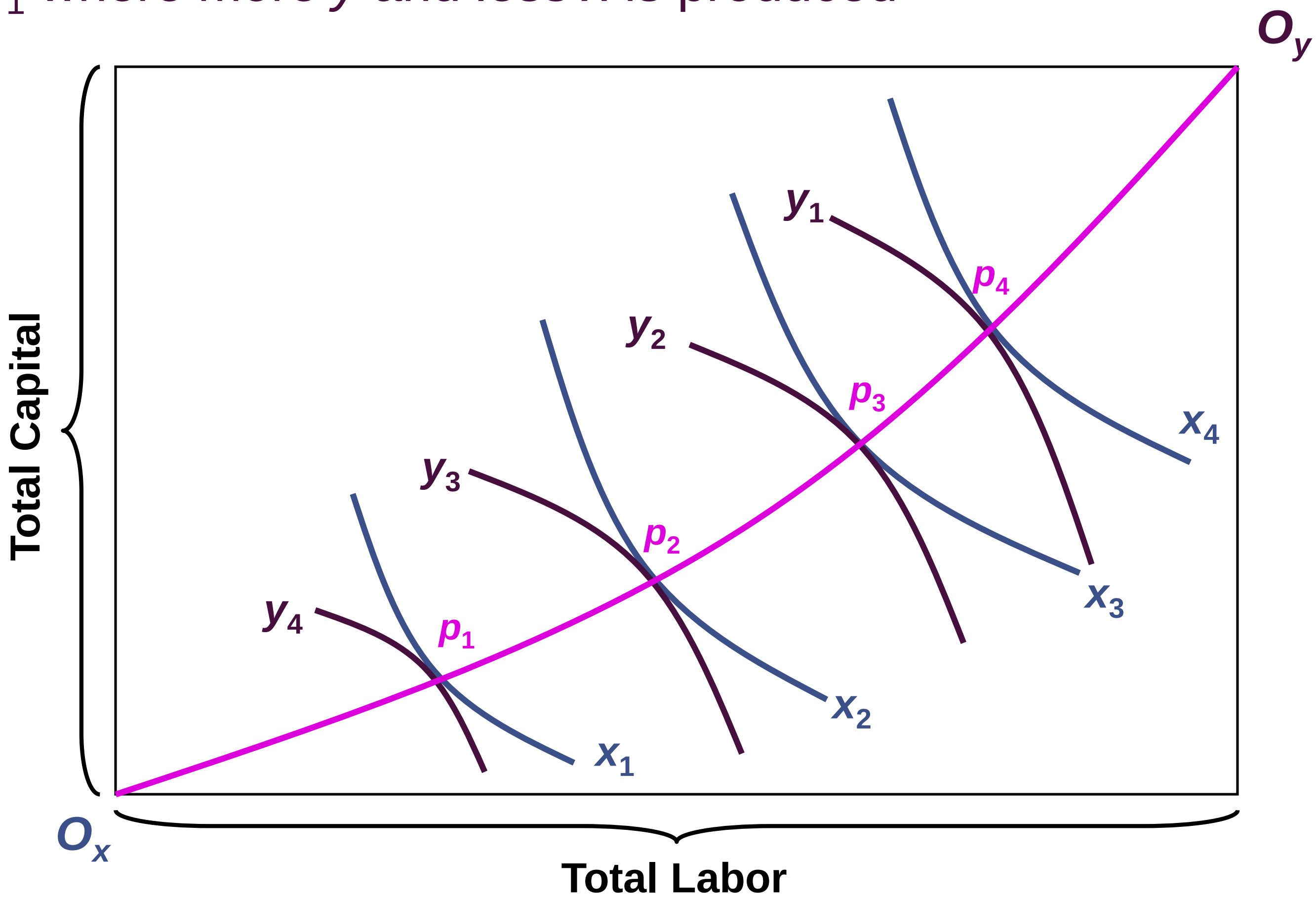


The Corn Laws Debate

- We can use an Edgeworth box diagram to see the effects of tariff reduction on the use of labor and capital
- If the corn laws were repealed, there would be an increase in the production of manufactured goods and a decline in the production of grain

The Corn Laws Debate

A repeal of the corn laws would result in a movement from p_3 to p_1 where more y and less x is produced



The Corn Laws Debate

- If we assume that grain production is relatively capital intensive, the movement from p_3 to p_1 causes the ratio of k to l to rise in both industries
  - the relative price of capital will fall
  - the relative price of labor will rise
- The repeal of the corn laws will be harmful to capital owners and helpful to laborers

Political Support for Trade Policies

- Trade policies may affect the relative incomes of various factors of production
- In the United States, exports tend to be intensive in their use of skilled labor whereas imports tend to be intensive in their use of unskilled labor
  - free trade policies will result in rising relative wages for skilled workers and in falling relative wages for unskilled workers

Existence of General Equilibrium Prices

- Beginning with 19th century investigations by Leon Walras, economists have examined whether there exists a set of prices that equilibrates all markets simultaneously
  - if this set of prices exists, how can it be found?

Existence of General Equilibrium Prices

- Suppose that there are n goods in fixed supply in this economy
  - let S_i ($i = 1, \ldots, n$) be the total supply of good i available
  - let p_i ($i = 1, \ldots n$) be the price of good i
- The total demand for good i depends on all prices

$$D_i(p_1, \ldots, p_n) \text{ for } i = 1, \ldots, n$$

Existence of General Equilibrium Prices

- We will write this demand function as dependent on the whole set of prices (P)

$$D_i(P)$$

- Walras' problem: Does there exist an equilibrium set of prices such that

$$D_i(P^*) = S_i$$

for all values of i?

Excess Demand (超额需求) Functions

- The excess demand function for any good *i* at any set of prices (*P*) is defined to be

$$ED_i(P) = D_i(P) - S_i$$

- This means that the equilibrium condition can be rewritten as

$$ED_i(P^*) = D_i(P^*) - S_i = 0$$

Excess Demand Functions

- Demand functions are homogeneous of degree zero
  - this implies that we can only establish equilibrium relative prices in a Walrasian-type model
- Walras also assumed that demand functions are continuous
  - small changes in price lead to small changes in quantity demanded

Walras' Law

- A final observation that Walras made was that the *n* excess demand equations are not independent of one another
- Walras' law shows that the total value of excess demand is zero at any set of prices

$$\sum_{i=1}^{n} P_i \cdot ED_i(P) = 0$$

Walras' Law

- Walras' law holds for any set of prices (not just equilibrium prices)
- There can be neither excess demand for all goods together nor excess supply

Walras' Proof of the Existence of Equilibrium Prices

- The market equilibrium conditions provide (n-1) independent equations in (n-1) unknown relative prices
  - can we solve the system for an equilibrium condition?
    - the equations are not necessarily linear
    - all prices must be nonnegative
- To attack these difficulties, Walras set up a complicated proof

Walras' Proof of the Existence of Equilibrium Prices

- Start with an arbitrary set of prices
- Holding the other n-1 prices constant, find the equilibrium price for good 1 (p_1')
- Holding p_1' and the other n-2 prices constant, solve for the equilibrium price of good 2 (p_2')
  - in changing p_2 from its initial position to p_2', the price calculated for good 1 does not need to remain an equilibrium price

Walras' Proof of the Existence of Equilibrium Prices

- Using the provisional prices p_1' and p_2', solve for p_3'
  - proceed in this way until an entire set of provisional relative prices has been found
- In the 2nd iteration of Walras' proof, p_2',…,p_n' are held constant while a new equilibrium price is calculated for good 1
  - proceed in this way until an entire new set of prices is found

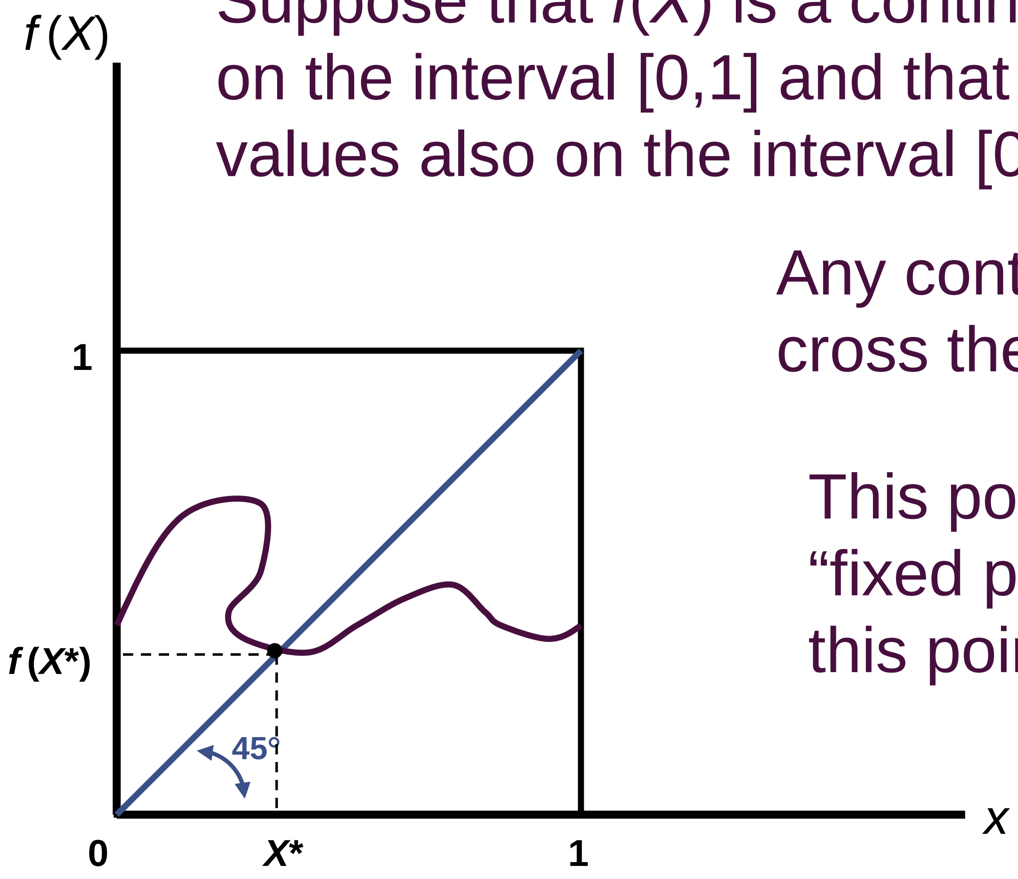
Walras' Proof of the Existence of Equilibrium Prices

- The importance of Walras' proof is its ability to demonstrate the simultaneous nature of the problem of finding equilibrium prices
- Because it is cumbersome, it is not generally used today
- More recent work uses some relatively simple tools from advanced mathematics

Brouwer's Fixed-Point Theorem

- Any continuous mapping [$F(X)$] of a closed, bounded, convex set into itself has at least one fixed point (X^*) such that $F(X^*) = X^*$

Brouwer's Fixed-Point Theorem



Suppose that *f*(*X*) is a continuous function defined on the interval [0,1] and that *f*(*X*) takes on the values also on the interval [0,1]

Any continuous function must cross the 45° line

This point of crossing is a "fixed point" because *f* maps this point (*X**) into itself

Brouwer's Fixed-Point Theorem

- A mapping is a rule that associates the points in one set with points in another set
  - let *X* be a point for which a mapping (*F*) is defined
    - the mapping associates *X* with some point $Y = F(X)$
  - if a mapping is defined over a subset of *n*-dimensional space (*S*), and if every point in *S* is associated (by the rule *F*) with some other point in *S*, the mapping is said to map *S* into itself

Brouwer's Fixed-Point Theorem

- A mapping is continuous if points that are "close" to each other are mapped into other points that are "close" to each other
- The Brouwer fixed-point theorem considers mappings defined on certain kinds of sets
  - closed (they contain their boundaries)
  - bounded (none of their dimensions is infinitely large)
  - convex (they have no "holes" in them)

Proof of the Existence of Equilibrium Prices

- Because only relative prices matter, it is convenient to assume that prices have been defined so that the sum of all prices is equal to 1
- Thus, for any arbitrary set of prices ($p_1,\ldots,p_n$), we can use normalized prices of the form

$$p_i' = \frac{p_i}{\sum_{i=1}^{n} p_i}$$

Proof of the Existence of Equilibrium Prices

- These new prices will retain their original relative values and will sum to 1

$$\frac{p_i'}{p_j'} = \frac{p_i}{p_j}$$

- These new prices will sum to 1

$$\sum_{i=1}^{n} p_i' = 1$$

Proof of the Existence of Equilibrium Prices

- We will assume that the feasible set of prices (*S*) is composed of all nonnegative numbers that sum to 1
  - *S* is the set to which we will apply Brouwer's theorem
  - *S* is closed, bounded, and convex
  - we will need to define a continuous mapping of *S* into itself

Free Goods

- Equilibrium does not really require that excess demand be zero for every market
- Goods may exist for which the markets are in equilibrium where supply exceeds demand (negative excess demand)
  - it is necessary for the prices of these goods to be equal to zero
  - “free goods”

Free Goods

- The equilibrium conditions are

$$ED_i\,(P^*) = 0 \text{ for } p_i^* > 0$$

$$ED_i\,(P^*) \leq 0 \text{ for } p_i^* = 0$$

- Note that this set of equilibrium prices continues to obey Walras' law

Mapping the Set of Prices Into Itself

- In order to achieve equilibrium, prices of goods in excess demand should be raised, whereas those in excess supply should have their prices lowered

Mapping the Set of Prices Into Itself

- We define the mapping $F(P)$ for any normalized set of prices (P), such that the ith component of $F(P)$ is given by

$$F^i(P) = p_i + ED_i(P)$$

- The mapping performs the necessary task of appropriately raising or lowering prices

Mapping the Set of Prices Into Itself

- Two problems exist with this mapping
- First, nothing ensures that the prices will be nonnegative
  - the mapping must be redefined to be

$$F^i(P) = \text{Max}\ [p_i + ED_i(P),0]$$

  - the new prices defined by the mapping must be positive or zero

Mapping the Set of Prices Into Itself

- Second, the recalculated prices are not necessarily normalized
  - they will not sum to 1
  - it will be simple to normalize such that

$$\sum_{i=1}^{n} F^i(P) = 1$$

  - we will assume that this normalization has been done

Application of Brouwer's Theorem

- Thus, *F* satisfies the conditions of the Brouwer fixed-point theorem
  - it is a continuous mapping of the set *S* into itself
- There exists a point (P^*) that is mapped into itself
- For this point,

$$p_i^* = \text{Max}\,[p_i^* + ED_i(P^*),0] \quad \text{for all } i$$

Application of Brouwer's Theorem

- This says that P^* is an equilibrium set of prices
  - for $p_i^* > 0$,

$$p_i^* = p_i^* + ED_i(P^*)$$
$$ED_i(P^*) = 0$$

  - For $p_i^* = 0$,

$$p_i^* + ED_i(P^*) \leq 0$$
$$ED_i(P^*) \leq 0$$

A General Equilibrium with Three Goods

- The economy of Oz is composed only of three precious metals: (1) silver, (2) gold, and (3) platinum
  - there are 10 (thousand) ounces of each metal available
- The demands for gold and platinum are

$$D_2 = -2\frac{p_2}{p_1} + \frac{p_3}{p_1} + 11 \qquad D_3 = -\frac{p_2}{p_1} - 2\frac{p_3}{p_1} + 18$$

A General Equilibrium with Three Goods

- Equilibrium in the gold and platinum markets requires that demand equal supply in both markets simultaneously

$$-2\frac{p_2}{p_1}+\frac{p_3}{p_1}+11=10$$

$$-\frac{p_2}{p_1}-2\frac{p_3}{p_1}+18=10$$

A General Equilibrium with Three Goods

- This system of simultaneous equations can be solved as

$$p_2/p_1 = 2 \qquad\qquad p_3/p_1 = 3$$

- In equilibrium:
  - gold will have a price twice that of silver
  - platinum will have a price three times that of silver
  - the price of platinum will be 1.5 times that of gold

A General Equilibrium with Three Goods

- Because Walras' law must hold, we know

$$p_1ED_1 = -p_2ED_2 - p_3ED_3$$

- Substituting the excess demand functions for gold and silver and substituting, we get

$$p_1ED_1 = 2\frac{p_2^2}{p_1} - \frac{p_2p_3}{p_1} - p_2 + \frac{p_2p_3}{p_1} + 2\frac{p_3^2}{p_1} - 8p_3$$

$$ED_1 = 2\frac{p_2^2}{p_1^2} + 2\frac{p_3^2}{p_1^2} - \frac{p_2}{p_1} - 8\frac{p_3}{p_1}$$

Smith's Invisible Hand Hypothesis

- Adam Smith believed that the competitive market system provided a powerful "invisible hand" that ensured resources would find their way to where they were most valued
- Reliance on the economic self-interest of individuals and firms would result in a desirable social outcome

Smith's Invisible Hand Hypothesis

- Smith's insights gave rise to modern welfare economics
- The "First Theorem of Welfare Economics" suggests that there is an exact correspondence between the efficient allocation of resources and the competitive pricing of these resources

Pareto Efficiency

- An allocation of resources is Pareto efficient if it is not possible (through further reallocations) to make one person better off without making someone else worse off
- The Pareto definition identifies allocations as being "inefficient" if unambiguous improvements are possible

Efficiency in Production

- An allocation of resources is efficient in production (or "technically efficient") if no further reallocation would permit more of one good to be produced without necessarily reducing the output of some other good
- Technical efficiency is a precondition for Pareto efficiency but does not guarantee Pareto efficiency

Efficient Choice of Inputs for a Single Firm

- A single firm with fixed inputs of labor and capital will have allocated these resources efficiently if they are fully employed and if the *RTS* between capital and labor is the same for every output the firm produces

Efficient Choice of Inputs for a Single Firm

- Assume that the firm produces two goods (x and y) and that the available levels of capital and labor are k' and l'
- The production function for x is given by

$$x = f(k_x, l_x)$$

- If we assume full employment, the production function for y is

$$y = g(k_y, l_y) = g(k' - k_x, l' - l_x)$$

Efficient Choice of Inputs for a Single Firm

- Technical efficiency requires that *x* output be as large as possible for any value of *y* (*y*')
- Setting up the Lagrangian and solving for the first-order conditions:

$$\mathbf{L} = f(k_x, l_x) + \lambda[y' - g(k' - k_x, l' - l_x)]$$

$$\partial \mathbf{L}/\partial k_x = f_k + \lambda g_k = 0$$

$$\partial \mathbf{L}/\partial l_x = f_l + \lambda g_l = 0$$

$$\partial \mathbf{L}/\partial \lambda = y' - g(k' - k_x, l' - l_x) = 0$$

Efficient Choice of Inputs for a Single Firm

- From the first two conditions, we can see that

$$\frac{f_k}{f_l} = \frac{g_k}{g_l}$$

- This implies that

$$RTS_x\ (k \text{ for } l) = RTS_y\ (k \text{ for } l)$$

Efficient Allocation of Resources among Firms

- Resources should be allocated to those firms where they can be most efficiently used
  - the marginal physical product of any resource in the production of a particular good should be the same across all firms that produce the good

Efficient Allocation of Resources among Firms

- Suppose that there are two firms producing *x* and their production functions are

$$f_1(k_1, l_1)$$

$$f_2(k_2, l_2)$$

- Assume that the total supplies of capital and labor are k' and l'

Efficient Allocation of Resources among Firms

- The allocational problem is to maximize

$$x = f_1(k_1, l_1) + f_2(k_2, l_2)$$

subject to the constraints

$$k_1 + k_2 = k'$$

$$l_1 + l_2 = l'$$

- Substituting, the maximization problem becomes

$$x = f_1(k_1, l_1) + f_2(k' - k_1, l' - l_1)$$

Efficient Allocation of Resources among Firms

- First-order conditions for a maximum are

$$\frac{\partial x}{\partial k_1} = \frac{\partial f_1}{\partial k_1} + \frac{\partial f_2}{\partial k_1} = \frac{\partial f_1}{\partial k_1} - \frac{\partial f_2}{\partial k_2} = 0$$

$$\frac{\partial x}{\partial l_1} = \frac{\partial f_1}{\partial l_1} + \frac{\partial f_2}{\partial l_1} = \frac{\partial f_1}{\partial l_1} - \frac{\partial f_2}{\partial l_2} = 0$$

Efficient Allocation of Resources among Firms

- These first-order conditions can be rewritten as

$$\frac{\partial f_1}{\partial k_1} = \frac{\partial f_2}{\partial k_2} \qquad \frac{\partial f_1}{\partial l_1} = \frac{\partial f_2}{\partial l_2}$$

- The marginal physical product of each input should be equal across the two firms

Efficient Choice of Output by Firms

- Suppose that there are two outputs (*x* and *y*) each produced by two firms
- The production possibility frontiers for these two firms are

$$y_i = f_i(x_i) \quad \text{for } i=1,2$$

- The overall optimization problem is to produce the maximum amount of *x* for any given level of *y* (y^*)

Efficient Choice of Output by Firms

- The Lagrangian for this problem is

$$\mathbf{L} = x_1 + x_2 + \lambda[y^* - f_1(x_1) - f_2(x_2)]$$

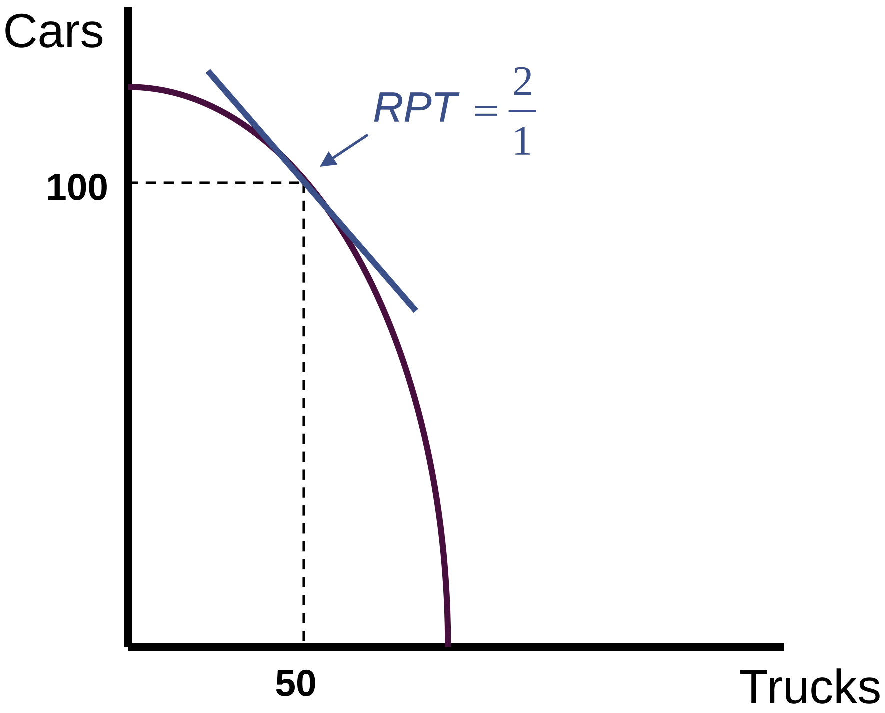
and yields the first-order condition:

$$\partial f_1/\partial x_1 = \partial f_2/\partial x_2$$

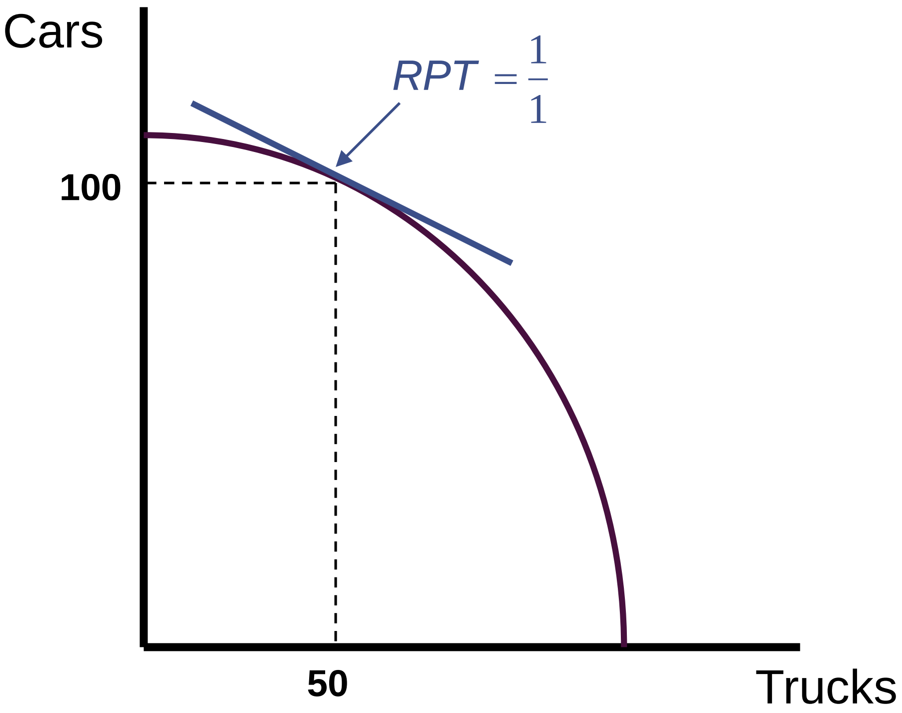
- The rate of product transformation (*RPT*) should be the same for all firms producing these goods

Efficient Choice of Output by Firms

Firm *A* is relatively efficient at producing cars, while Firm *B* is relatively efficient at producing trucks



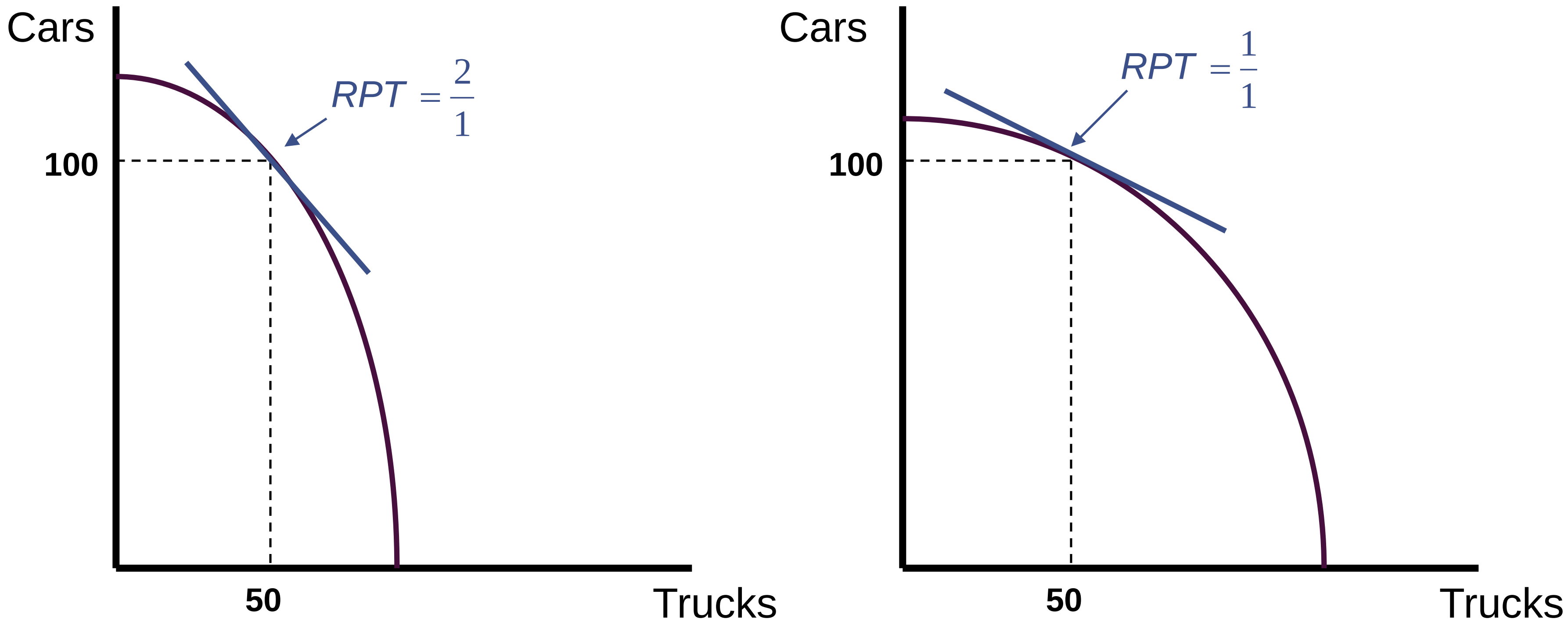
Firm *A*



Firm *B*

Efficient Choice of Output by Firms

If each firm was to specialize in its efficient product, total output could be increased



Firm *A* **Firm *B***

Theory of Comparative Advantage

- The theory of comparative advantage was first proposed by Ricardo
  - countries should specialize in producing those goods of which they are relatively more efficient producers
    - these countries should then trade with the rest of the world to obtain needed commodities
  - if countries do specialize this way, total world production will be greater

Efficiency in Product Mix

- Technical efficiency is not a sufficient condition for Pareto efficiency
  - demand must also be brought into the picture
- In order to ensure Pareto efficiency, we must be able to tie individual’s preferences and production possibilities together

Efficiency in Product Mix

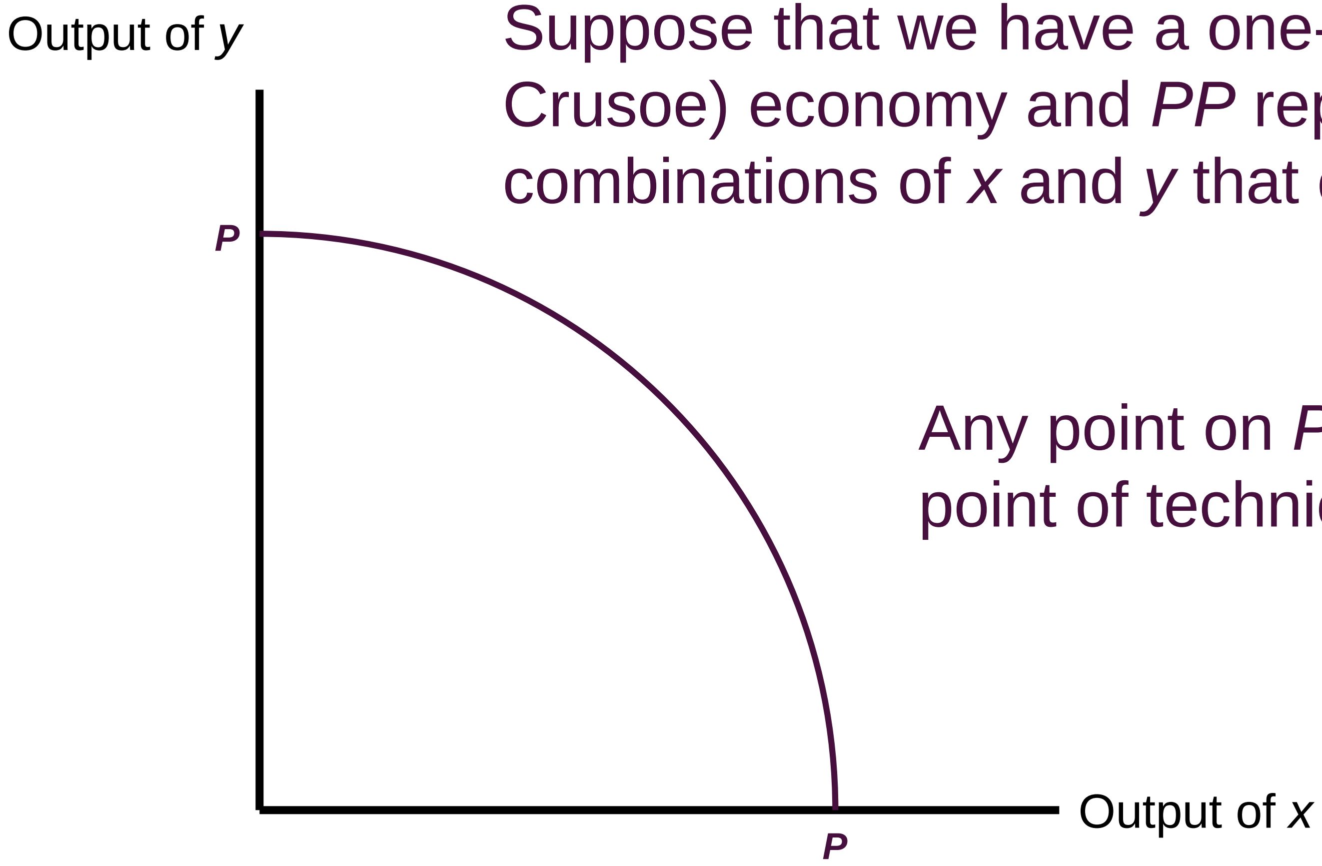
- The condition necessary to ensure that the right goods are produced is

$$MRS = RPT$$

  – the psychological rate of trade-off between the two goods in people's preferences must be equal to the rate at which they can be traded off in production

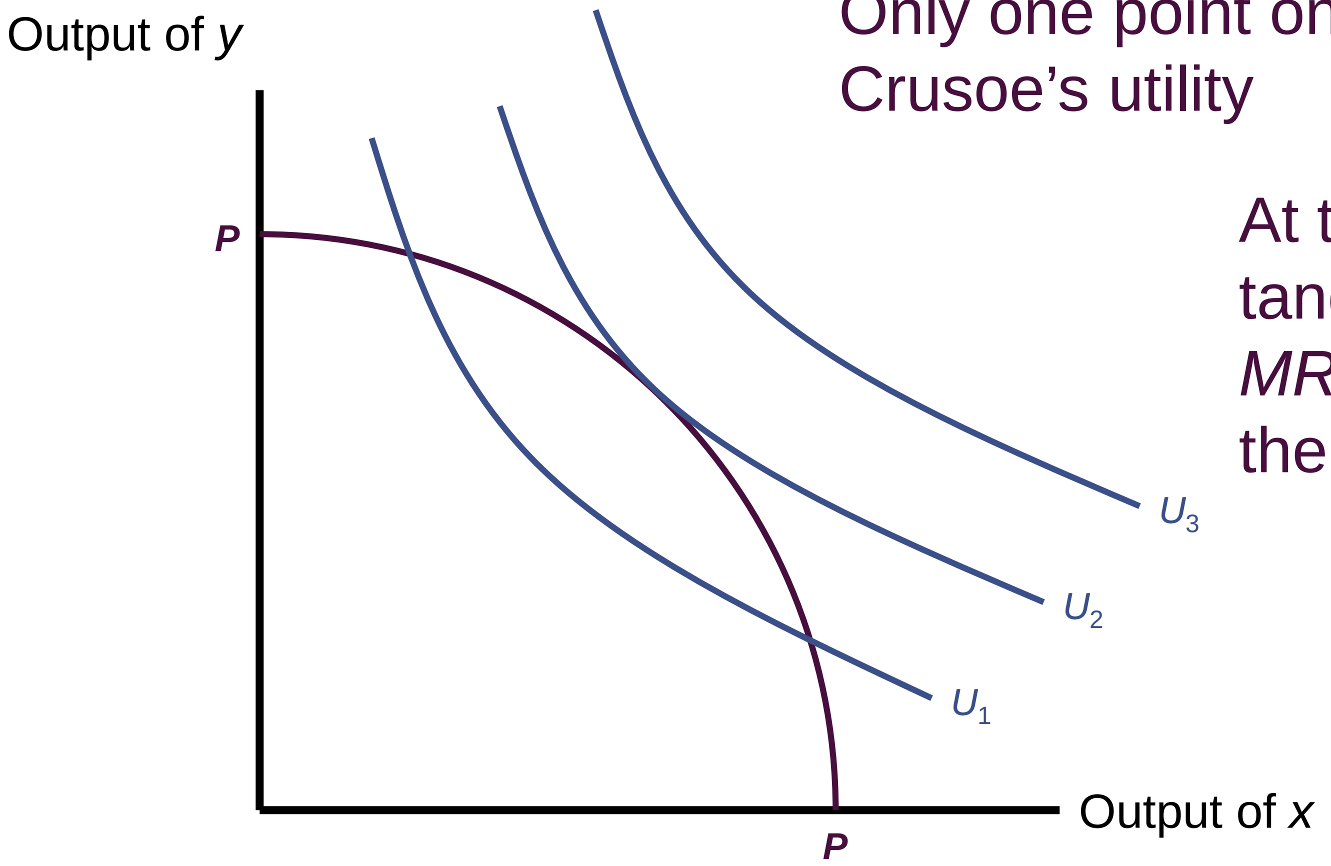
Efficiency in Product Mix

Suppose that we have a one-person (Robinson Crusoe) economy and *PP* represents the combinations of *x* and *y* that can be produced



Any point on *PP* represents a point of technical efficiency

Efficiency in Product Mix



Only one point on *PP* will maximize Crusoe's utility

At the point of tangency, Crusoe's *MRS* will be equal to the technical *RPT*

Efficiency in Product Mix

- Assume that there are only two goods (x and y) and one individual in society (Robinson Crusoe)
- Crusoe's utility function is

$$U = U(x,y)$$

- The production possibility frontier is

$$T(x,y) = 0$$

Efficiency in Product Mix

- Crusoe's problem is to maximize his utility subject to the production constraint

- Setting up the Lagrangian yields

$$\mathbf{L} = U(x,y) + \lambda[T(x,y)]$$

Efficiency in Product Mix

- First-order conditions for an interior maximum are

$$\frac{\partial \mathbf{L}}{\partial x} = \frac{\partial U}{\partial x} + \lambda \frac{\partial T}{\partial x} = 0$$

$$\frac{\partial \mathbf{L}}{\partial y} = \frac{\partial U}{\partial y} + \lambda \frac{\partial T}{\partial y} = 0$$

$$\frac{\partial \mathbf{L}}{\partial \lambda} = T(x, y) = 0$$

Efficiency in Product Mix

- Combining the first two, we get

$$\frac{\partial U / \partial x}{\partial U / \partial y} = \frac{\partial T / \partial x}{\partial T / \partial y}$$

or

$$MRS\ (x \text{ for } y) = -\frac{dy}{dx}(\text{along } T) = RPT\ (x \text{ for } y)$$

Competitive Prices and Efficiency

- Attaining a Pareto efficient allocation of resources requires that the rate of trade-off between any two goods be the same for all economic agents
- In a perfectly competitive economy, the ratio of the prices of the two goods provides the common rate of trade-off to which all agents will adjust

Competitive Prices and Efficiency

- Because all agents face the same prices, all trade-off rates will be equalized and an efficient allocation will be achieved
- This is the “First Theorem of Welfare Economics”

Efficiency in Production

- In minimizing costs, a firm will equate the *RTS* between any two inputs (*k* and *l*) to the ratio of their competitive prices (*w*/*v*)
  - this is true for all outputs the firm produces
  - *RTS* will be equal across all outputs

Efficiency in Production

- A profit-maximizing firm will hire additional units of an input (l) up to the point at which its marginal contribution to revenues is equal to the marginal cost of hiring the input (w)

$$p_x f_l = w$$

Efficiency in Production

- If this is true for every firm, then with a competitive labor market

$$p_x f_l^1 = w = p_x f_l^2$$

$$f_l^1 = f_l^2$$

- Every firm that produces *x* has identical marginal productivities of every input in the production of *x*

Efficiency in Production

- Recall that the *RPT* (of *x* for *y*) is equal to MC_x / MC_y
- In perfect competition, each profit-maximizing firm will produce the output level for which marginal cost is equal to price
- Since $p_x = MC_x$ and $p_y = MC_y$ for every firm, $RTS = MC_x / MC_y = p_x / p_y$

Efficiency in Production

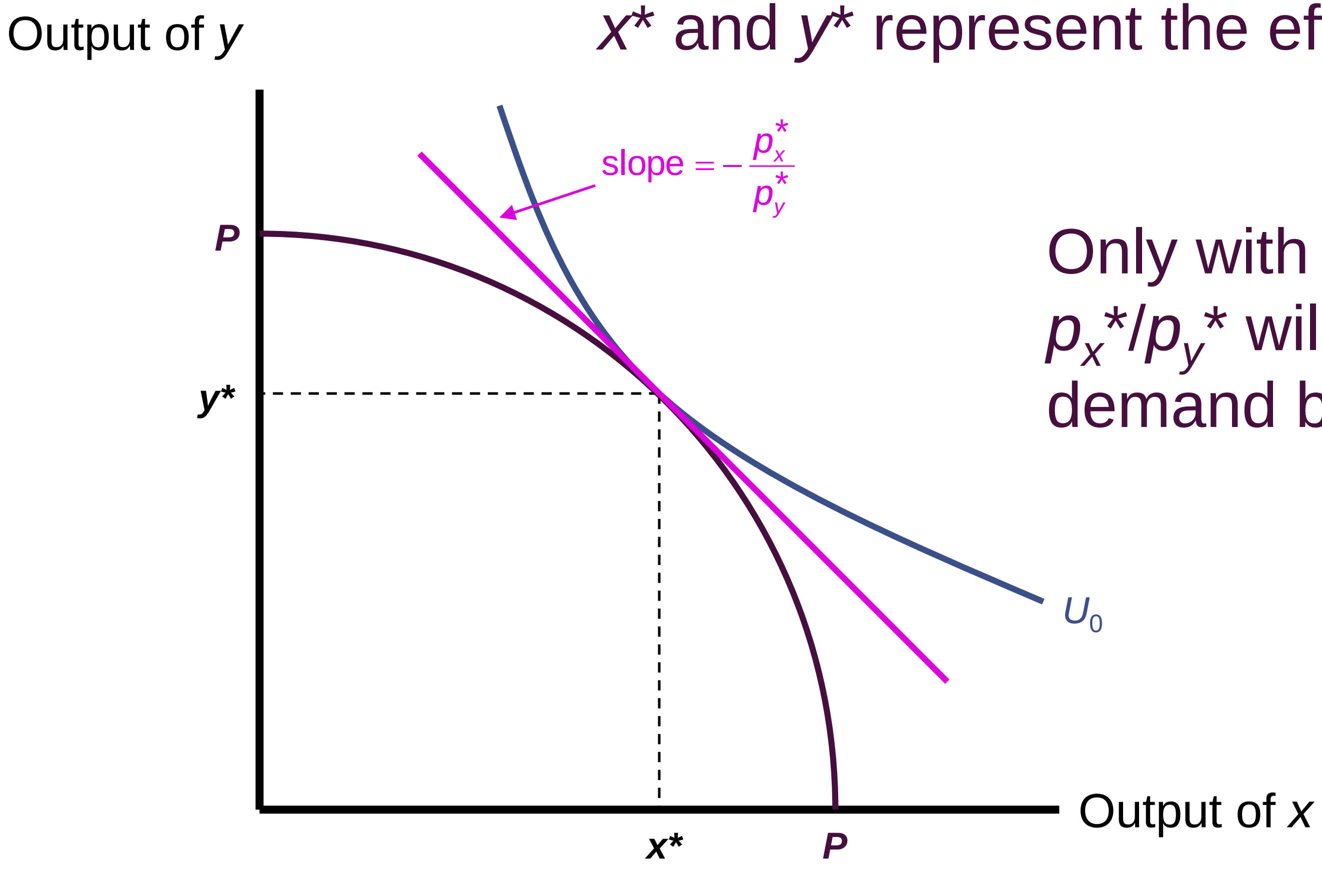
- Thus, the profit-maximizing decisions of many firms can achieve technical efficiency in production without any central direction
- Competitive market prices act as signals to unify the multitude of decisions that firms make into one coherent, efficient pattern

Efficiency in Product Mix

- The price ratios quoted to consumers are the same ratios the market presents to firms
- This implies that the *MRS* shared by all individuals will be equal to the *RPT* shared by all the firms
- An efficient mix of goods will therefore be produced

Efficiency in Product Mix

x^* and y^* represent the efficient output mix



Only with a price ratio of p_x^*/p_y^* will supply and demand be in equilibrium

Laissez-Faire Policies

- The correspondence between competitive equilibrium and Pareto efficiency provides some support for the laissez-faire position taken by many economists
  - government intervention may only result in a loss of Pareto efficiency

Departing from the Competitive Assumptions

- The ability of competitive markets to achieve efficiency may be impaired because of
  - imperfect competition
  - externalities
  - public goods
  - imperfect information

Imperfect Competition

- Imperfect competition includes all situations in which economic agents exert some market power in determining market prices
  - these agents will take these effects into account in their decisions
- Market prices no longer carry the informational content required to achieve Pareto efficiency

Externalities

- An externality occurs when there are interactions among firms and individuals that are not adequately reflected in market prices
- With externalities, market prices no longer reflect all of a good's costs of production
  - there is a divergence between private and social marginal cost

Public Goods

- Public goods have two properties that make them unsuitable for production in markets
  - they are nonrival
    - additional people can consume the benefits of these goods at zero cost
  - they are nonexclusive
    - extra individuals cannot be precluded from consuming the good

Imperfect Information

- If economic actors are uncertain about prices or if markets cannot reach equilibrium, there is no reason to expect that the efficiency property of competitive pricing will be retained

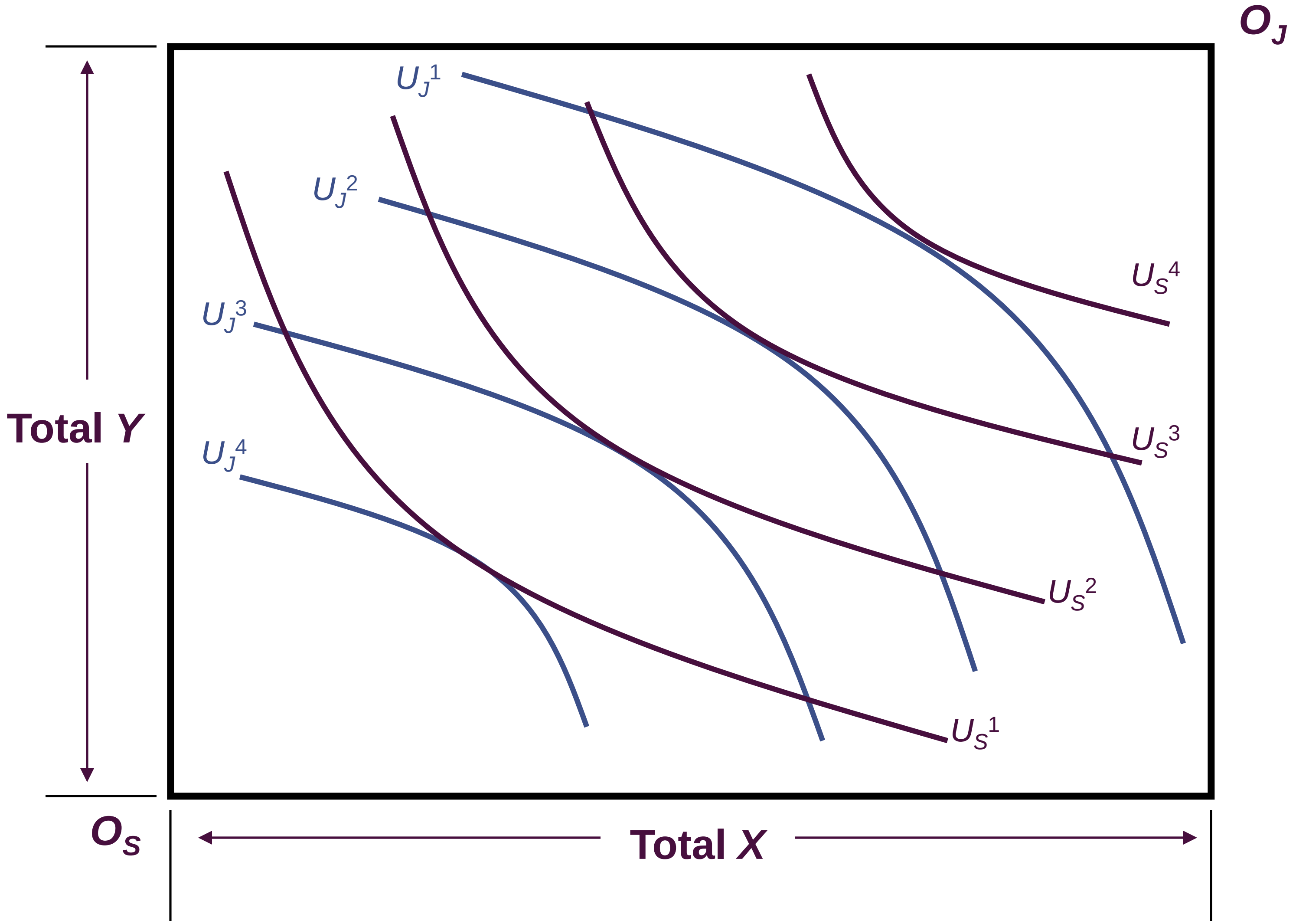
Distribution

- Although the First Theorem of Welfare Economics ensures that competitive markets will achieve efficient allocations, there are no guarantees that these allocations will exhibit desirable distributions of welfare among individuals

Distribution

- Assume that there are only two people in society (Smith and Jones)
- The quantities of two goods (x and y) to be distributed among these two people are fixed in supply
- We can use an Edgeworth box diagram to show all possible allocations of these goods between Smith and Jones

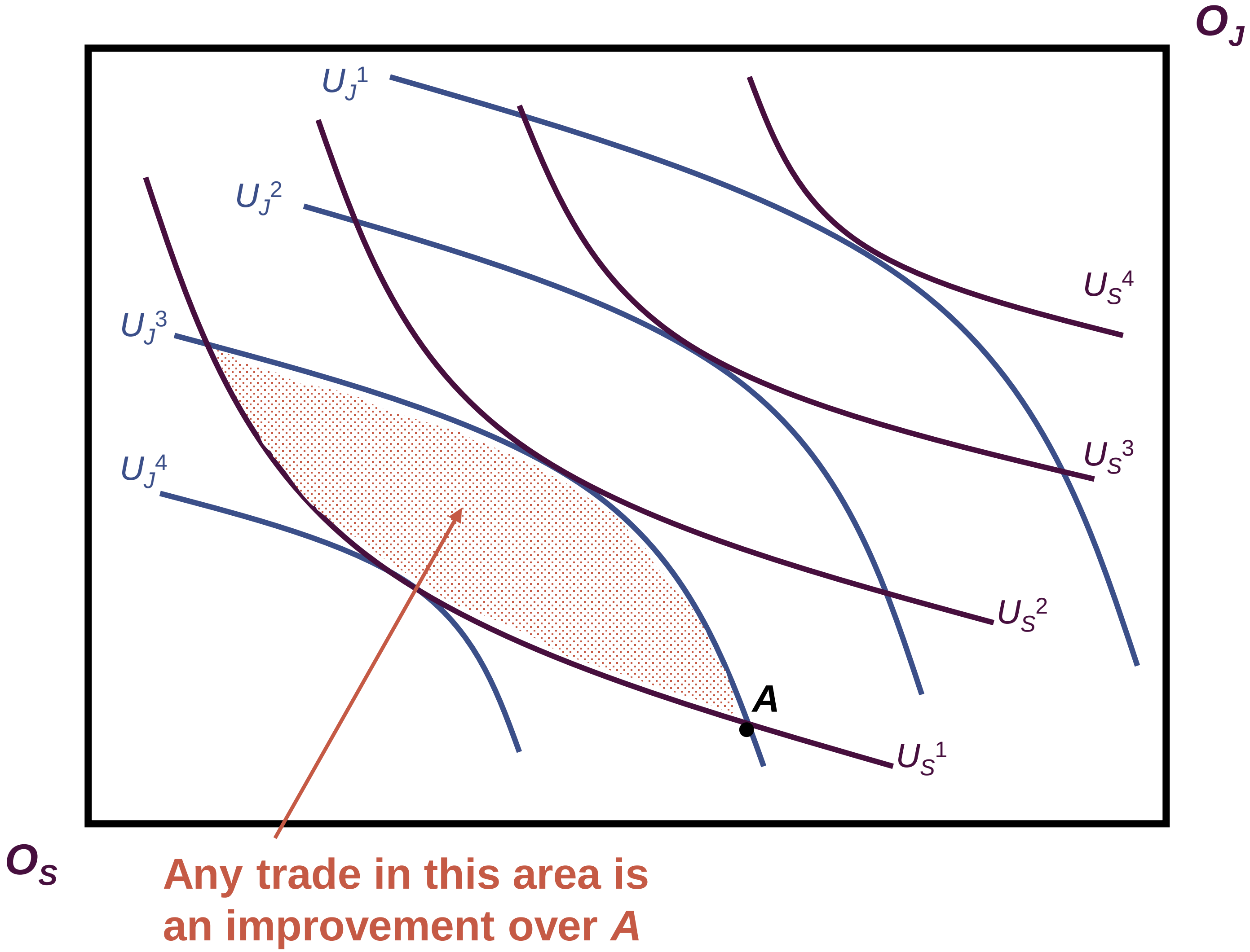
Distribution



Distribution

- Any point within the Edgeworth box in which the *MRS* for Smith is unequal to that for Jones offers an opportunity for Pareto improvements
  - both can move to higher levels of utility through trade

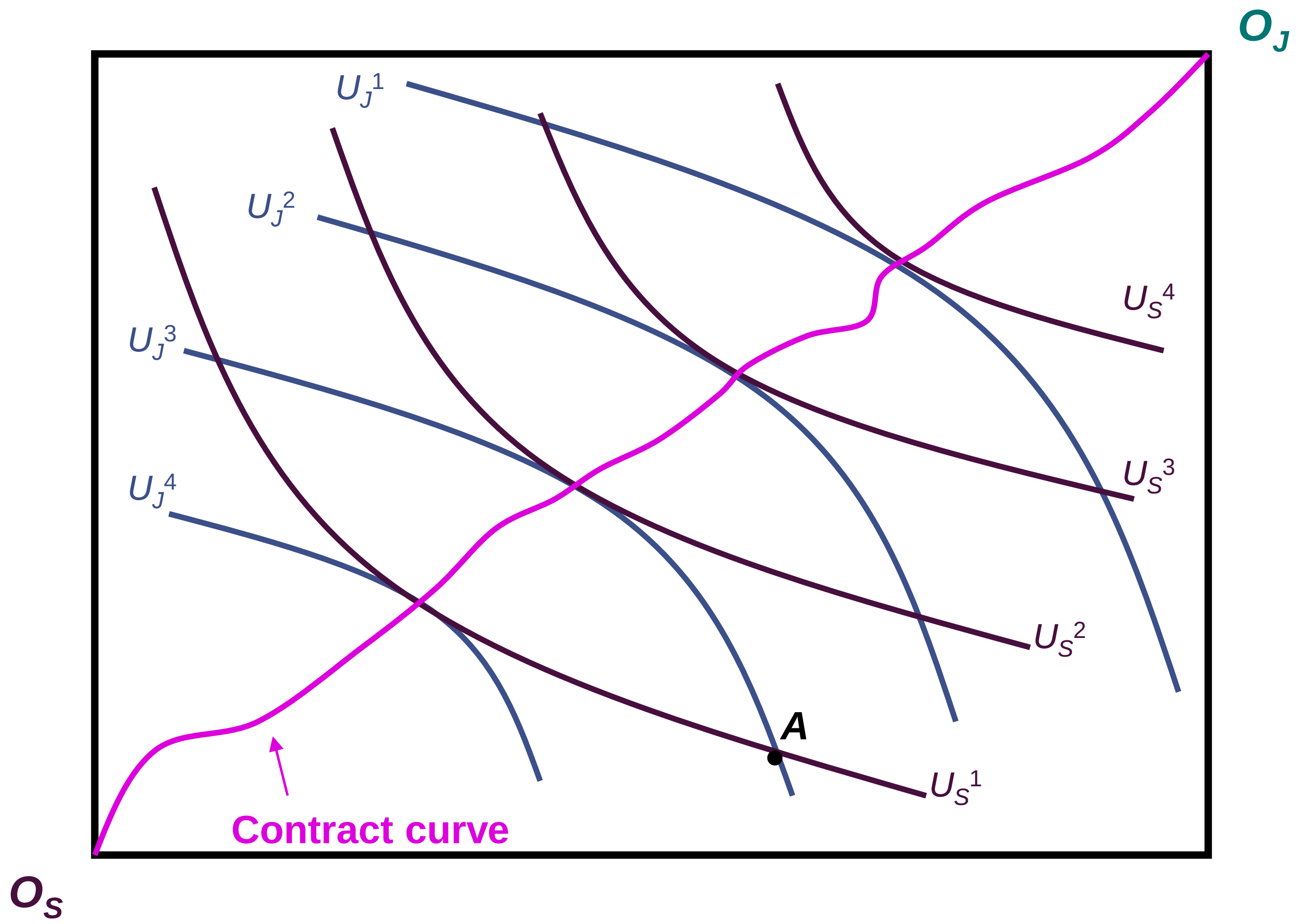
Distribution



Contract Curve

- In an exchange economy, all efficient allocations lie along a <u>contract curve</u>
  - points off the curve are necessarily inefficient
    - individuals can be made better off by moving to the curve
- Along the contract curve, individuals' preferences are rivals
  - one may be made better off only by making the other worse off

Contract Curve



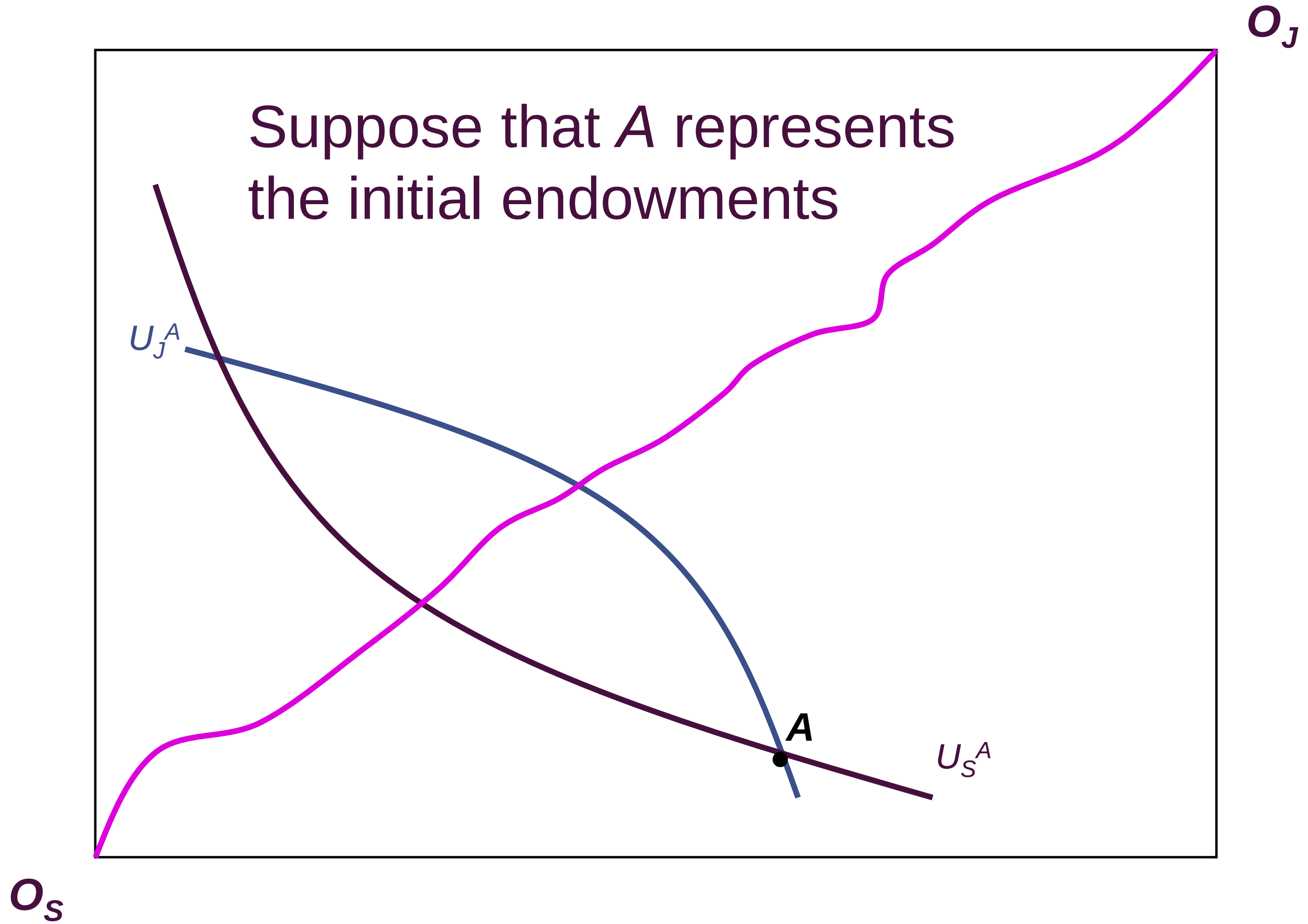
Exchange with Initial Endowments

- Suppose that the two individuals possess different quantities of the two goods at the start
  - it is possible that the two individuals could both benefit from trade if the initial allocations were inefficient

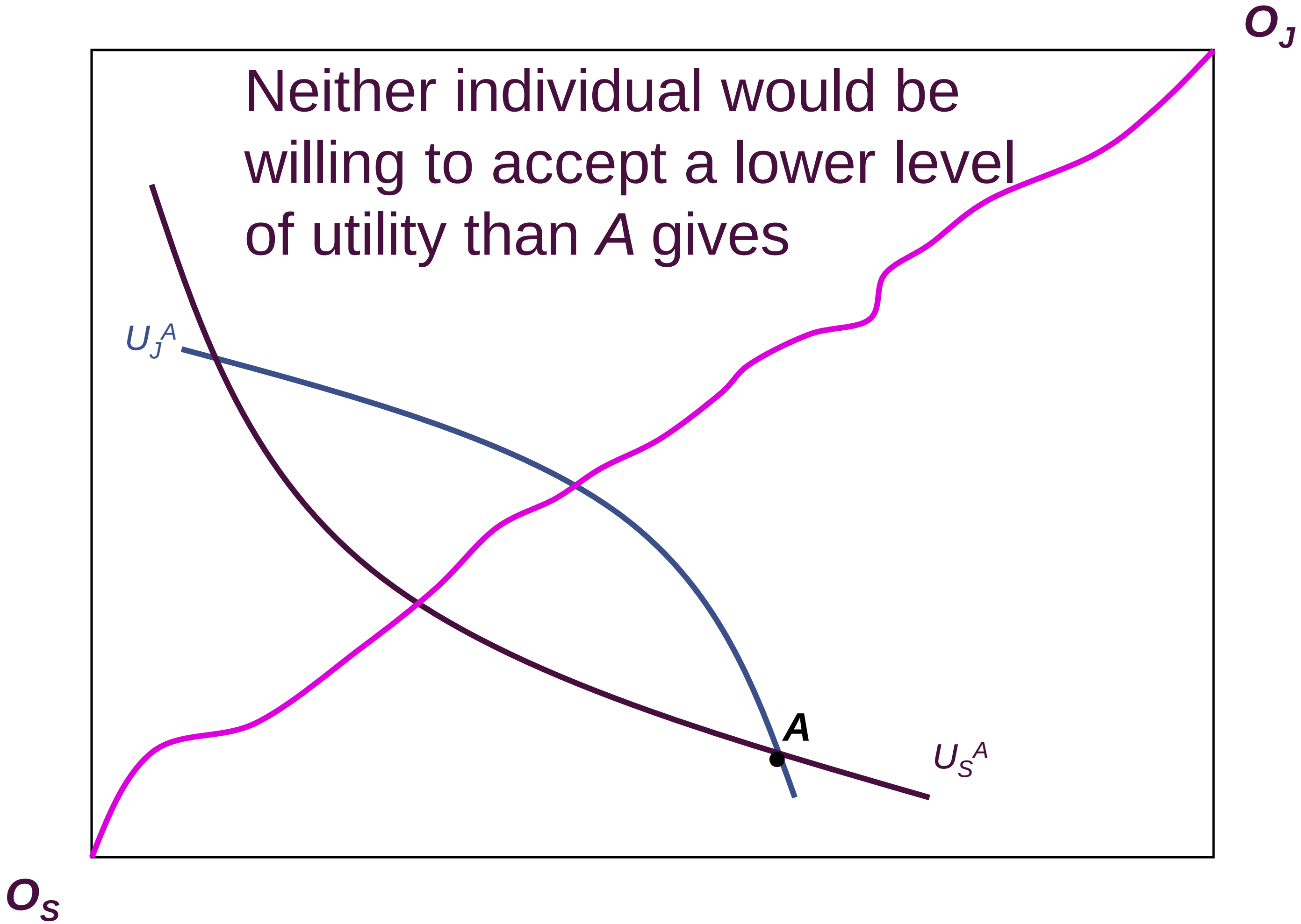
Exchange with Initial Endowments

- Neither person would engage in a trade that would leave him worse off
- Only a portion of the contract curve shows allocations that may result from voluntary exchange

Exchange with Initial Endowments

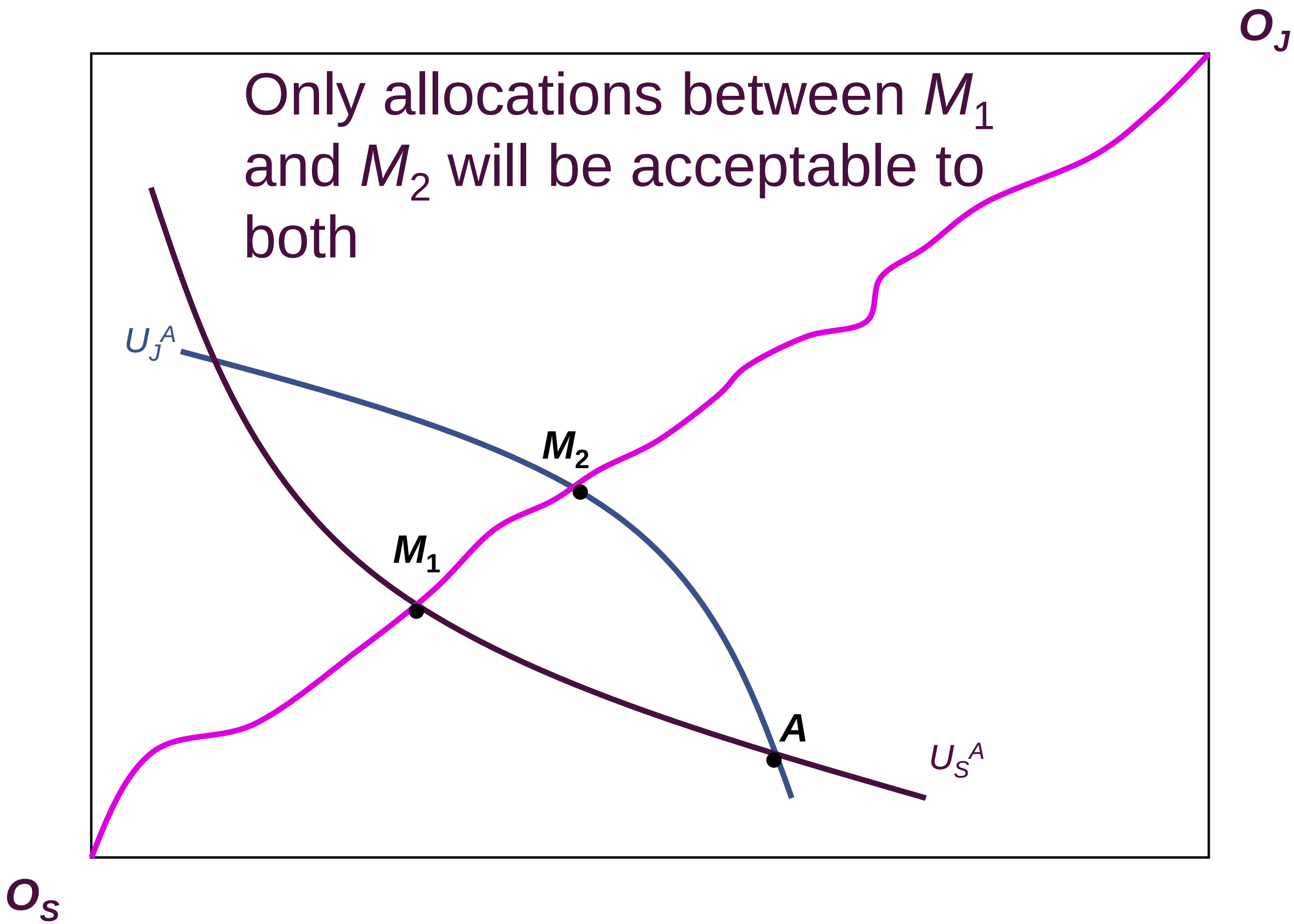


Exchange with Initial Endowments



Exchange with Initial Endowments

Only allocations between M_1 and M_2 will be acceptable to both



The Distributional Dilemma

- If the initial endowments are skewed in favor of some economic actors, the Pareto efficient allocations promised by the competitive price system will also tend to favor those actors
  - voluntary transactions cannot overcome large differences in initial endowments
  - some sort of transfers will be needed to attain more equal results

The Distributional Dilemma

- These thoughts lead to the "Second Theorem of Welfare Economics"
  - any desired distribution of welfare among individuals in an economy can be achieved in an efficient manner through competitive pricing if initial endowments are adjusted appropriately

Important Points to Note:

- Preferences and production technologies provide the building blocks upon which all general equilibrium models are based
  - one particularly simple version of such a model uses individual preferences for two goods together with a concave production possibility frontier for those two goods

Important Points to Note:

- Competitive markets can establish equilibrium prices by making marginal adjustments in prices in response to information about the demand and supply for individual goods
  - Walras' law ties markets together so that such a solution is assured (in most cases)

Important Points to Note:

- Competitive prices will result in a Pareto-efficient allocation of resources
  - this is the First Theorem of Welfare Economics

Important Points to Note:

- Factors that will interfere with competitive markets' abilities to achieve efficiency include
  - market power
  - externalities
  - existence of public goods
  - imperfect information

Important Points to Note:

- Competitive markets need not yield equitable distributions of resources, especially when initial endowments are very skewed
  - in theory any desired distribution can be attained through competitive markets accompanied by lump-sum transfers
    - there are many practical problems in implementing such transfers