Chapter 15

GAME THEORY MODELS OF PRICING

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Game Theory

- Game theory involves the study of strategic situations
- Game theory models attempt to portray complex strategic situations in a highly simplified and stylized setting
  - abstract from personal and institutional details in order to arrive at a representation of the situation that is mathematically tractable

Game Theory

- All games have three elements
  - players
  - strategies
  - payoffs
- Games may be cooperative or noncooperative

Players

- Each decision-maker in a game is called a player
  - can be an individual, a firm, an entire nation
- Each player has the ability to choose among a set of possible actions
- The specific identity of the players is irrelevant
  - no “good guys” or “bad guys”

Strategies

- Each course of action open to a player is called a strategy
- Strategies can be very simple or very complex
  - each is assumed to be well-defined
- In noncooperative games, players are uncertain about the strategies used by other players

Payoffs

- The final returns to the players at the end of the game are called <u>payoffs</u>
- Payoffs are usually measured in terms of utility
  - monetary payoffs are also used
- It is assumed that players can rank the payoffs associated with a game

Notation

- We will denote a game G between two players (A and B) by

$$G[S_A, S_B, U_A(a,b), U_B(a,b)]$$

where

S_A = strategies available for player A ($a \subset S_A$)

S_B = strategies available for player B ($b \subset S_B$)

U_A = utility obtained by player A when particular strategies are chosen

U_B = utility obtained by player B when particular strategies are chosen

Nash Equilibrium in Games

- At market equilibrium, no participant has an incentive to change his behavior
- In games, a pair of strategies (a^*,b^*) is defined to be a Nash equilibrium if a^* is player *A*'s best strategy when player *B* plays b^*, and b^* is player *B*'s best strategy when player *A* plays a^*

Nash Equilibrium in Games

- A pair of strategies (a^*,b^*) is defined to be a Nash equilibrium if

$$U_A(a^*,b^*) \geq U_A(a',b^*) \text{ for all } a' \subset S_A$$

$$U_B(a^*,b^*) \geq U_b(a^*,b') \text{ for all b}' \subset S_B$$

Nash Equilibrium in Games

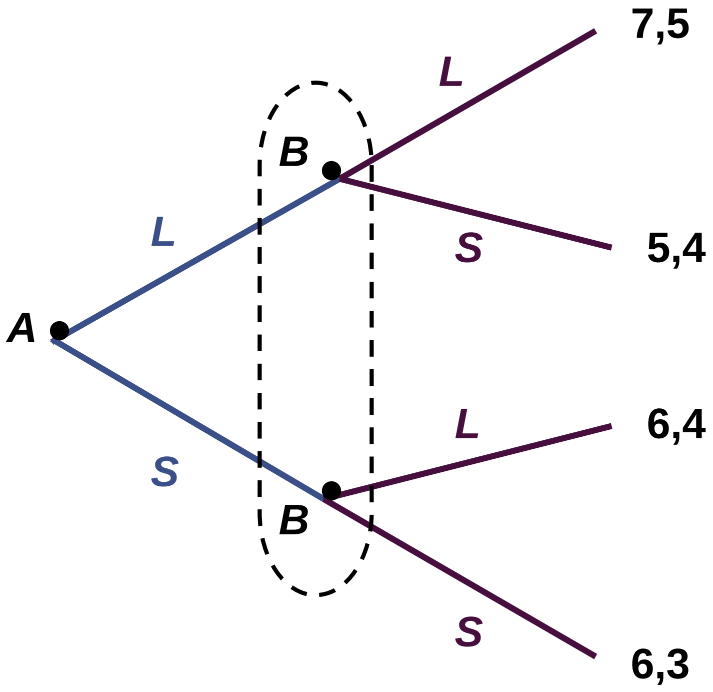
- If one of the players reveals the equilibrium strategy he will use, the other player cannot benefit
  - this is not the case with nonequilibrium strategies
- Not every game has a Nash equilibrium pair of strategies
- Some games may have multiple equilibria

A Dormitory Game

- Suppose that there are two students who must decide how loudly to play their stereos in a dorm
  - each may choose to play it loudly (*L*) or softly (*S*)

A Dormitory Game

A chooses loud (*L*) or soft (*S*)



B makes a similar choice

Neither player knows the other's strategy

Payoffs are in terms of *A*'s utility level and *B*'s utility level

A Dormitory Game

- Sometimes it is more convenient to describe games in tabular ("normal") form

		B's Strategies	
		L	***S***
A's Strategies	***L***	7,5	5,4
	S	6,4	6,3

A Dormitory Game

- A loud-play strategy is a dominant strategy for player *B*
  - the *L* strategy provides greater utility to *B* than does the *S* strategy no matter what strategy *A* chooses
- Player *A* will recognize that *B* has such a dominant strategy
  - *A* will choose the strategy that does the best against *B*'s choice of *L*

A Dormitory Game

- This means that *A* will also choose to play music loudly
- The *A*:*L*,*B*:*L* strategy choice obeys the criterion for a Nash equilibrium
  - because *L* is a dominant strategy for *B*, it is the best choice no matter what *A* does
  - if *A* knows that *B* will follow his best strategy, then *L* is the best choice for *A*

Existence of Nash Equilibria

- A Nash equilibrium is not always present in two-person games
- This means that one must explore the details of each game situation to determine whether such an equilibrium (or multiple equilibria) exists

No Nash Equilibria

- Any strategy is unstable because it offers the other players an incentive to adopt another strategy

		***B*'s Strategies**		
		Rock	Paper	Scissors
***A*'s Strategies**	Rock	0,0	1,-1	-1,1
	Paper	-1,1	0,0	1,-1
	Scissors	1,-1	-1,1	0,0

Two Nash Equilibria

- Both of the joint vacations represent Nash equilibria

		***B*'s Strategies**	
		Mountain	Seaside
***A*'s Strategies**	Mountain	2,1	0,0
	Seaside	0,0	1,2

Existence of Nash Equilibria

- There are certain types of two-person games in which a Nash equilibrium must exist
  - games in which the participants have a large number of strategies
    - games in which the strategies chosen by *A* and *B* are alternate levels of a single continuous variable
    - games where players use <u>mixed strategies</u>

Existence of Nash Equilibria

- In a game where players are permitted to use mixed strategies, each player may play the pure strategies with certain, pre-selected probabilities
  - player *A* may flip a coin to determine whether to play music loudly or softly
  - the possibility of playing the pure strategies with any probabilities a player may choose, converts the game into one with an infinite number of mixed strategies

The Prisoners' Dilemma

- The most famous two-person game with an undesirable Nash equilibrium outcome

		***B*'s Strategies**	
		Confess	Not Confess
***A*'s Strategies**	Confess	***A***: 3 years ***B***: 3 years	***A***: 6 months ***B***: 10 years
	Not Confess	***A***: 10 years ***B***: 6 months	***A***: 2 years ***B***: 2 years

The Prisoners' Dilemma

- An ironclad agreement by both prisoners not to confess will give them the lowest amount of joint jail time
  - this solution is not stable
- The "confess" strategy dominates for both *A* and *B*
  - these strategies constitute a Nash equilibrium

The Tragedy of the Common

- This example is used to signify the environmental problems of overuse that occur when scarce resources are treated as "common property"
- Assume that two herders are deciding how many of their yaks they should let graze on the village common
  – problem: the common is small and can rapidly become overgrazed

The Tragedy of the Common

- Suppose that the per yak value of grazing on the common is

$$V(Y_A, Y_B) = 200 - (Y_A + Y_B)^2$$

  where Y_A and Y_B = number of yaks of each herder

- Note that both $V_i < 0$ and $V_{ii} < 0$
  - an extra yak reduces V and this marginal effect increases with additional grazing

The Tragedy of the Common

- Solving herder *A*'s value maximization problem:

$$\text{Max } Y_A V = \text{Max } [200Y_A - Y_A(Y_A + Y_B)^2]$$

- The first-order condition is

$$200 - 2Y_A^2 - 2Y_AY_B - Y_A^2 - 2Y_AY_B - Y_B^2$$
$$= 200 - 3Y_A^2 - 4Y_AY_B - Y_B^2 = 0$$

- Similarly, for *B* the optimal strategy is

$$200 - 3Y_B^2 - 4Y_BY_A - Y_A^2 = 0$$

The Tragedy of the Common

- For a Nash equilibrium, the values for Y_A and Y_B must solve both of these conditions
- Using the symmetry condition $Y_A = Y_B$

$$200 = 8Y_A^2 = 8Y_B^2$$

$$Y_A = Y_B = 5$$

- Each herder will obtain 500 [= $5\cdot(200-10^2)$] in return
- Given this choice, neither herder has an incentive to change his behavior

The Tragedy of the Common

- The Nash equilibrium is not the best use of the common
- $Y_A = Y_B = 4$ provides greater return to each herder $[4 \cdot (200 - 8^2) = 544]$
- But $Y_A = Y_B = 4$ is not a stable equilibrium
  - if *A* announces that $Y_A = 4$, *B* will have an incentive to increase Y_B
  - there is an incentive to cheat

Cooperation and Repetition

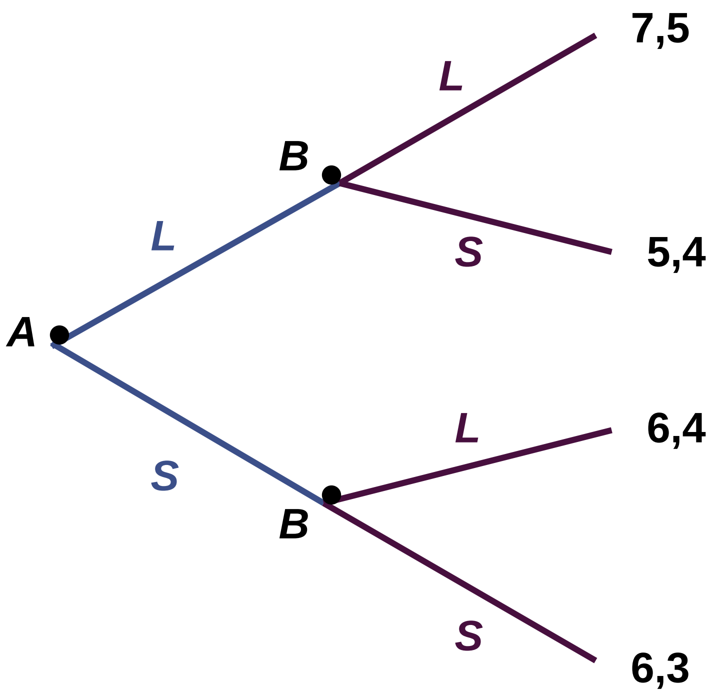
- Cooperation among players can result in outcomes that are preferred to the Nash outcome by both players
  - the cooperative outcome is unstable because it is not a Nash equilibrium
- Repeated play may foster cooperation

A Two-Period Dormitory Game

- Let's assume that *A* chooses his decibel level first and then *B* makes his choice
- In effect, that means that the game has become a two-period game
  - *B*'s strategic choices must take into account the information available at the start of period two

A Two-Period Dormitory Game

A chooses loud (*L*) or soft (*S*)



B makes a similar choice knowing *A*'s choice

Thus, we should put *B*'s strategies in a form that takes the information on *A*'s choice into account

A Two-Period Dormitory Game

- Each strategy is stated as a pair of actions showing what *B* will do depending on *A*'s actions

		***B*'s Strategies**			
		L,L	***L,S***	***S,L***	***S,S***
***A*'s Strategies**	***L***	7,5	7,5	5,4	5,4
	S	6,4	6,3	6,4	6,3

A Two-Period Dormitory Game

- There are 3 Nash equilibria in this game
  - *A*:*L*, *B*:(*L*,*L*)
  - *A*:*L*, *B*:(*L*,*S*)
  - *A*:*S*, *B*:(*S*,*L*)

_B_'s Strategies

_A_'s Strategies	***L,L***	***L,S***	***S,L***	***S,S***
L	7,5	7,5	5,4	5,4
S	6,4	6,3	6,4	6,3

A Two-Period Dormitory Game

- *A*:*L*, *B*:(*L*,*S*) and *A*:*S*, *B*:(*S*,*L*) are implausible
  - each incorporates a noncredible threat on the part of *B*

		***B*'s Strategies**			
		L*,*L	***L*,*S***	***S*,*L***	***S*,*S***
***A*'s Strategies**	***L***	7,5	7,5	5,4	5,4
	S	6,4	6,3	6,4	6,3

A Two-Period Dormitory Game

- Thus, the game is reduced to the original payoff matrix where (*L*,*L*) is a dominant strategy for *B*
  - *A* will recognize this and will always choose *L*
- This is a subgame perfect equilibrium
  - a Nash equilibrium in which the strategy choices of each player do not involve noncredible threats

Subgame Perfect Equilibrium

- A “subgame” is the portion of a larger game that begins at one decision node and includes all future actions stemming from that node
- To qualify to be a subgame perfect equilibrium, a strategy must be a Nash equilibrium in each subgame of a larger game

Repeated Games

- Many economic situations can be modeled as games that are played repeatedly
  - consumers' regular purchases from a particular retailer
  - firms' day-to-day competition for customers
  - workers' attempts to outwit their supervisors

Repeated Games

- An important aspect of a repeated game is the expanded strategy sets that become available to the players
  - opens the way for credible threats and subgame perfection

Repeated Games

- The number of repetitions is also important
  - in games with a fixed, finite number of repetitions, there is little room for the development of innovative strategies
  - games that are played an infinite number of times offer a much wider array of options

Prisoners' Dilemma Finite Game (skipped)

- If the game was played only once, the Nash equilibrium *A*:*U*, *B*:*L* would be the expected outcome

_B_'s Strategies

		L	***R***
_A_'s Strategies	***U***	1,1	3,0
	D	0,3	2,2

Prisoners' Dilemma Finite Game

- This outcome is inferior to *A*:*D*, *B*:*R* for each player

		***B*'s Strategies**	
		L	***R***
***A*'s Strategies**	***U***	1,1	3,0
	D	0,3	2,2

Prisoners' Dilemma Finite Game

- Suppose this game is to be repeatedly played for a finite number of periods (*T*)
- Any expanded strategy in which *A* promises to play *D* in the final period is not credible
  – when *T* arrives, *A* will choose strategy *U*
- The same logic applies to player *B*

Prisoners' Dilemma Finite Game

- Any subgame perfect equilibrium for this game can only consist of the Nash equilibrium strategies in the final round
  - *A*:*U*,*B*:*L*
- The logic that applies to period *T* also applies to period *T*-1
- The only subgame perfect equilibrium in this finite game is to require the Nash equilibrium in every round

Game with Infinite Repetitions

- In this case, each player can announce a “trigger strategy”
  - promise to play the cooperative strategy as long as the other player does
  - when one player deviates from the pattern, the game reverts to the repeating single-period Nash equilibrium

Game with Infinite Repetitions

- Whether the twin trigger strategy represents a subgame perfect equilibrium depends on whether the promise to play cooperatively is credible
- Suppose that *A* announces that he will continue to play the trigger strategy by playing cooperatively in period *K*

Game with Infinite Repetitions

- If *B* decides to play cooperatively, payoffs of 2 can be expected to continue indefinitely
- If *B* decides to cheat, the payoff in period *K* will be 3, but will fall to 1 in all future periods
  - the Nash equilibrium will reassert itself

Game with Infinite Repetitions

- If δ is player *B*'s discount rate, the present value of continued cooperation is

$$2 + \delta 2 + \delta^2 2 + \ldots = 2/(1-\delta)$$

- The payoff from cheating is

$$3 + \delta 1 + \delta^2 1 + \ldots = 3 + 1/(1-\delta)$$

- Continued cooperation will be credible if

$$2/(1-\delta) > 3 + 1/(1-\delta)$$

$$\delta > 1/2$$

The Tragedy of the Common Revisited (skipped)

- The overgrazing of yaks on the village common may not persist in an infinitely repeated game
- Assume that each herder has only two strategies available
  - bringing 4 or 5 yaks to the common
- The Nash equilibrium (*A*:5,*B*:5) is inferior to the cooperative outcome (*A*:4,*B*:4)

The Tragedy of the Common Revisited

- With an infinite number of repetitions, both players would find it attractive to adopt cooperative trigger strategies if

$$544/(1-\delta) > 595 + 500(1-\delta)$$

$$\delta > 551/595 = 0.93$$

Pricing in Static Games

- Suppose there are only two firms (*A* and *B*) producing the same good at a constant marginal cost (*c*)
  - the strategies for each firm consist of choosing prices (P_A and P_B) subject only to the condition that the firm's price must exceed *c*
- Payoffs in the game will be determined by demand conditions

Pricing in Static Games

- Because output is homogeneous and marginal costs are constant, the firm with the lower price will gain the entire market
- If $P_A = P_B$, we will assume that the firms will share the market equally

Pricing in Static Games

- In this model, the only Nash equilibrium is $P_A = P_B = c$
  - if firm *A* chooses a price greater than *c*, the profit-maximizing response for firm *B* is to choose a price slightly lower than P_A and corner the entire market
  - but *B*'s price (if it exceeds *c*) cannot be a Nash equilibrium because it provides firm *A* with incentive for further price cutting

Pricing in Static Games

- Therefore, only by choosing $P_A = P_B = c$ will the two firms have achieved a Nash equilibrium
  - we end up with a competitive solution even though there are only two firms
- This pricing strategy is sometimes referred to as a Bertrand equilibrium

Pricing in Static Games

- The Bertrand result depends crucially on the assumptions underlying the model
  - if firms do not have equal costs or if the goods produced by the two firms are not perfect substitutes, the competitive result no longer holds

Pricing in Static Games

- Other duopoly models that depart from the Bertrand result treat price competition as only the final stage of a two-stage game in which the first stage involves various types of entry or investment considerations for the firms

Pricing in Static Games

- Consider the case of two owners of natural springs who are deciding how much water to supply
- Assume that each firm must choose a certain capacity output level
  - marginal costs are constant up to that level and infinite thereafter

Pricing in Static Games

- A two-stage game where firms choose capacity first (and then price) is formally identical to the Cournot analysis
  - the quantities chosen in the Cournot equilibrium represent a Nash equilibrium
    - each firm correctly perceives what the other's output will be
  - once the capacity decisions are made, the only price that can prevail is that for which quantity demanded is equal to total capacity

Pricing in Static Games

- Suppose that capacities are given by q_A' and q_B' and that

$$P' = D^{-1}(q_A' + q_B')$$

where D^{-1} is the inverse demand function

- A situation in which $P_A = P_B < P'$ is not a Nash equilibrium
  - total quantity demanded > total capacity so one firm could increase its profits by raising its price and still sell its capacity

Pricing in Static Games

- Likewise, a situation in which $P_A = P_B > P'$ is not a Nash equilibrium
  - total quantity demanded < total capacity so at least one firm is selling less than its capacity
    - by cutting price, this firm could increase its profits by taking all possible sales up to its capacity
    - the other firm would end up lowering its price as well

Pricing in Static Games

- The only Nash equilibrium that will prevail is $P_A = P_B = P'$
  - this price will fall short of the monopoly price but will exceed marginal cost
- The results of this two-stage game are indistinguishable from the Cournot model

Pricing in Static Games

- The Bertrand model predicts competitive outcomes in a duopoly situation
- The Cournot model predicts monopoly-like inefficiencies
- This suggests that actual behavior in duopoly markets may exhibit a wide variety of outcomes depending on the way in which competition occurs

Repeated Games and Tacit Collusion (skipped)

- Players in infinitely repeated games may be able to adopt subgame-perfect Nash equilibrium strategies that yield better outcomes than simply repeating a less favorable Nash equilibrium indefinitely
  - do the firms in a duopoly have to endure the Bertrand equilibrium forever?
  - can they achieve more profitable outcomes through tacit collusion?

Repeated Games and Tacit Collusion

- With any finite number of replications, the Bertrand result will remain unchanged
  - any strategy in which firm *A* chooses $P_A > c$ in period *T* (the final period) offers *B* the option of choosing $P_A > P_B > c$
    - *A*'s threat to charge P_A in period *T* is noncredible
  - a similar argument applies to any period prior to *T*

Repeated Games and Tacit Collusion

- If the pricing game is repeated over infinitely many periods, twin "trigger" strategies become feasible
  - each firm sets its price equal to the monopoly price (P_M) providing the other firm did the same in the prior period
  - if the other firm "cheated" in the prior period, the firm will opt for competitive pricing in all future periods

Repeated Games and Tacit Collusion

- Suppose that, after the pricing game has been proceeding for several periods, firm *B* is considering cheating
  - by choosing $P_B < P_A = P_M$ it can obtain almost all of the single period monopoly profits (π_M)

Repeated Games and Tacit Collusion

- If firm *B* continues to collude tacitly with *A*, it will earn its share of the profit stream

$$(\pi_M + \delta\pi_M + \delta^2\pi_M + \ldots + \delta^n\pi_M + \ldots)/2$$

$$= (\pi_M/2)[1/(1- \delta)]$$

where δ is the discount factor applied to future profits

Repeated Games and Tacit Collusion

- Cheating will be unprofitable if

$$\pi_M < (\pi_M /2)[1/(1- \delta)]$$

or if

$$\delta > 1/2$$

- Providing that firms are not too impatient, the trigger strategies represent a subgame perfect Nash equilibrium of tacit collusion

Tacit Collusion

- Suppose only two firms produce steel bars for jailhouse windows
- Bars are produced at a constant *AC* and *MC* of $10 and the demand for bars is

$$Q = 5{,}000 - 100P$$

- Under Bertrand competition, each firm will charge a price of $10 and a total of 4,000 bars will be sold

Tacit Collusion

- The monopoly price in this market is $30
  - each firm has an incentive to collude
  - total monopoly profits will be $40,000 each period (each firm will receive $20,000)
  - any one firm will consider a next-period price cut only if $40,000 > $20,000 $(1/1-\delta)$
    - δ will have to be fairly high for this to occur

Tacit Collusion

- The viability of a trigger price strategy may depend on the number of firms
  - suppose there are 8 producers
  - total monopoly profits will be $40,000 each period (each firm will receive $5,000)
  - any one firm will consider a next-period price cut if $40,000 > $5,000 (1/1-$\delta$)
    - this is likely at reasonable levels of δ

Generalizations and Limitations

- The viability of tacit collusion in game theory models is very sensitive to the assumptions made
- We assumed that:
  - firm *B* can easily detect that firm *A* has cheated
  - firm *B* responds to cheating by adopting a harsh response that not only punishes *A*, but also condemns *B* to zero profits forever

Generalizations and Limitations

- In more general models of tacit collusion, these assumptions can be relaxed
  - difficulty in monitoring other firm’s behavior
  - other forms of punishment
  - differentiated products

Entry, Exit, and Strategy

- In previous models, we have assumed that entry and exit are driven by the relationship between the prevailing market price and a firm's average cost
- The entry and exit issue can become considerably more complex

Entry, Exit, and Strategy

- A firm wishing to enter or exit a market must make some conjecture about how its actions will affect the future market price
  - – this requires the firm to consider what its rivals will do
  - – this may involve a number of strategic ploys
    - especially when a firm's information about its rivals is imperfect

Sunk Costs and Commitment(skipped)

- Many game theoretic models of entry stress the importance of a firm's commitment to a specific market
  - large capital investments that cannot be shifted to another market will lead to a large level of commitment on the part of the firm

Sunk Costs and Commitment

- Sunk costs are one-time investments that must be made to enter a market
  - these allow the firm to produce in the market but have no residual value if the firm leaves the market
  - could include expenditures on unique types of equipment or job-specific training of workers

First-Mover Advantage in Cournot's Natural Springs

- Under the Stackelberg version of this model, each firm has two possible strategies
  - be a leader ($q_i = 60$)
  - be a follower ($q_i = 30$)

First-Mover Advantage in Cournot's Natural Springs

- The payoffs for these two strategies are:

		B's Strategies	
		Leader (q_B = 60)	Follower (q_B = 30)
A's Strategies	Leader (q_A = 60)	*A*: 0 *B*: 0	*A*: $1,800 *B*: $ 900
	Follower (q_A = 30)	*A*: $ 900 *B*: $1,800	*A*: $1,600 *B*: $1,600

First-Mover Advantage in Cournot's Natural Springs

- The leader-leader strategy for each firm proves to be disastrous
  - it is not a Nash equilibrium
    - if firm *A* knows that firm *B* will adopt a leader strategy, its best move is to be a follower
- A follower-follower choice is profitable for both firms
  - this choice is unstable because it gives each firm an incentive to cheat

First-Mover Advantage in Cournot's Natural Springs

- With simultaneous moves, either of the leader-follower pairs represents a Nash equilibrium
- But if one firm has the opportunity to move first, it can dictate which of the two equilibria is chosen
  – this is the first-mover advantage

Entry Deterrence(skipped)

- In some cases, first-mover advantages may be large enough to deter all entry by rivals
  - however, it may not always be in the firm's best interest to create that large a capacity

Entry Deterrence

- With economies of scale, the possibility for profitable entry deterrence is increased
  - if the first mover can adopt a large-enough scale of operation, it may be able to limit the scale of a potential entrant
    - the potential entrant will experience such high average costs that there would be no advantage to entering the market

Entry Deterrence in Cournot's Natural Spring

- Assume that each spring owner must pay a fixed cost of operations ($784)
- The Nash equilibrium leader-follower strategies remain profitable for both firms
  - if firm *A* moves first and adopts the leader's role, *B*'s profits are relatively small ($116)
    - *A* could push *B* out of the market by being a bit more aggressive

Entry Deterrence in Cournot's Natural Spring

- Since *B*'s reaction function is unaffected by the fixed costs, firm *A* knows that

$$q_B = (120 - q_A)/2$$

and market price is given by

$$P = 120 - q_A - q_B$$

- Firm *A* knows that *B*'s profits are

$$\pi_B = Pq_B - 784$$

Entry Deterrence in Cournot's Natural Spring

- When *B* is a follower, its profits depend only on q_A
- Therefore,

$$\pi_B = \left(\frac{120 - q_A}{2}\right)^2 - 784$$

- Firm *A* can ensure nonpositive profits for firm *B* by choosing $q_A \geq 64$
  - Firm A will earn profits of $2,800

Limit Pricing(skipped)

- Are there situations where a monopoly might purposely choose a low ("limit") price policy to deter entry into its market?
- In most simple situations, the limit pricing strategy does not yield maximum profits and is not sustainable over time
  - choosing $P_L < P_M$ will only deter entry if P_L is lower than the *AC* of any potential entrant

Limit Pricing

- If the monopoly and the potential entrant have the same costs, the only limit price sustainable is $P_L = AC$
  - defeats the purpose of being a monopoly because $\pi = 0$
- Thus, the basic monopoly model offers little room for entry deterrence through pricing behavior

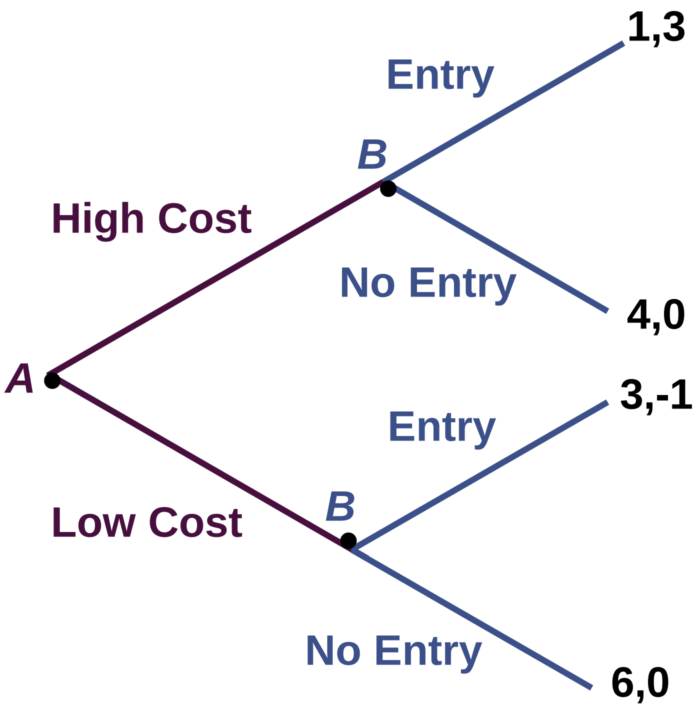
Limit Pricing and Incomplete Information

- Believable models of limit pricing must depart from traditional assumptions
- The most important set of such models involves incomplete information
  - if an incumbent monopolist knows more about the market situation than a potential entrant, the monopolist may be able to deter entry

Limit Pricing and Incomplete Information

- Suppose that an incumbent monopolist may have either “high” or “low” production costs as a result of past decisions
- The profitability of firm *B*’s entry into the market depends on *A*’s costs
- We can use a tree diagram to show *B*’s dilemma

Limit Pricing and Incomplete Information



The profitability of entry for Firm *B* depends on Firm *A*'s costs which are unknown to *B*

Limit Pricing and Incomplete Information

- Firm *B* could use whatever information it has to develop a subjective probability of *A*'s cost structure
- If *B* assumes that there is a probability of ρ that *A* has high cost and $(1-\rho)$ that it has low cost, entry will yield positive expected profits if

$$E(\pi_B) = \rho(3) + (1-\rho)(-1) > 0$$

$$\rho > 1/4$$

Limit Pricing and Incomplete Information

- Regardless of its true costs, firm *A* is better off if *B* does not enter
- One way to ensure this is for *A* to convince *B* that $\rho < 1/4$
- Firm *A* may choose a low-price strategy then to signal firm *B* that its costs are low
  - this provides a possible rationale for limit pricing

Predatory Pricing

- The structure of many models of predatory behavior is similar to that used in limit pricing models
  - stress incomplete information
- A firm wishes to encourage its rival to exit the market
  - it takes actions to affect its rival's views of the future profitability of remaining in the market

Games of Incomplete Information

- Each player in a game may be one of a number of possible types (t_A and t_B)
  - player types can vary along several dimensions
- We will assume that our player types have differing potential payoff functions
  - each player knows his own payoff but does not know his opponent's payoff with certainty

Games of Incomplete Information

- Each player's conjectures about the opponent's player type are represented by belief functions [$f_A(t_B)$]
  - consist of the player's probability estimates of the likelihood that his opponent is of various types
- Games of incomplete information are sometimes referred to as Bayesian games

Games of Incomplete Information

- We can now generalize the notation for the game

$$G[S_A, S_B, t_A, t_B, f_A, f_B, U_A(a,b,t_A,t_B), U_B(a,b,t_A,t_B)]$$

- The payoffs to *A* and *B* depend on the strategies chosen ($a \subset S_A$, $b \subset S_B$) and the player types

Games of Incomplete Information

- For one-period games, it is fairly easy to generalize the Nash equilibrium concept to reflect incomplete information
  - we must use expected utility because each player’s payoffs depend on the unknown player type of the opponent

Games of Incomplete Information

- A strategy pair (a^*,b^*) will be a Bayesian-Nash equilibrium if a^* maximizes A's expected utility when B plays b^* and vice versa

$$E[U_A(a^*,b^*,t_A,t_B)] = \sum_{t_B} f_A(t_B)U(a^*,b^*,t_A,t_B)$$

$$\geq E[U_B(a',b^*,t_A,t_B)] \text{ for all } a' \subset S_A$$

$$E[U_A(a^*,b^*,t_A,t_B)] = \sum_{t_A} f_B(t_A)U(a^*,b^*,t_A,t_B)$$

$$\geq E[U_B(a^*,b',t_A,t_B)] \text{ for all } b' \subset S_B$$

A Bayesian-Cournot Equilibrium

- Suppose duopolists compete in a market for which demand is given by

$$P = 100 - q_A - q_B$$

- Suppose that $MC_A = MC_B = 10$
  - the Nash (Cournot) equilibrium is $q_A = q_B = 30$ and payoffs are $\pi_A = \pi_B = 900$

A Bayesian-Cournot Equilibrium

- Suppose that $MC_A = 10$, but MC_B may be either high (= 16) or low (= 4)
- Suppose that *A* assigns equal probabilities to these two "types" for *B* so that the expected $MC_B = 10$
- *B* does not have to consider expectations because it knows there is only a single *A* type

A Bayesian-Cournot Equilibrium

- B chooses q_B to maximize

$$\pi_B = (P - MC_B)(q_B) = (100 - MC_B - q_A - q_B)(q_B)$$

- The first-order condition for a maximum is

$$q_B^* = (100 - MC_B - q_A)/2$$

- Depending on MC_B, this is either

$$q_B^* = (84 - q_A)/2 \quad \text{or}$$

$$q_B^* = (96 - q_A)/2$$

A Bayesian-Cournot Equilibrium

- Firm *A* must take into account that *B* could face either high or low marginal costs so its expected profit is

$$\pi_A = 0.5(100 - MC_A - q_A - q_{BH})(q_A) + 0.5(100 - MC_A - q_A - q_{BL})(q_A)$$

$$\pi_A = (90 - q_A - 0.5q_{BH} - 0.5q_{BL})(q_A)$$

A Bayesian-Cournot Equilibrium

- The first-order condition for a maximum is

$$q_A^* = (90 - 0.5q_{BH} - 0.5q_{BL})/2$$

- The Bayesian-Nash equilibrium is:

$$q_A^* = 30$$
$$q_{BH}^* = 27$$
$$q_{BL}^* = 33$$

- These choices represent an *ex ante* equilibrium

Mechanism Design and Auctions(skipped)

- The concept of Bayesian-Nash equilibrium has been used to study auctions
  - by examining equilibrium solutions under various possible auction rules, game theorists have devised procedures that obtain desirable results
    - achieving high prices for the goods being sold
    - ensuring the goods end up with those who value them most

An Oil Tract Auction

- Suppose two firms are bidding on a tract of land that may have oil underground
- Each firm has decided on a potential value for the tract (V_A and V_B)
- The seller would like to obtain the largest price possible for the land
  - the larger of V_A or V_B
- Will a simple sealed bid auction work?

An Oil Tract Auction

- To model this as a Bayesian game, we need to model each firm's beliefs about the other's valuations
  - $0 \leq V_i \leq 1$
  - each firm assumes that all possible values for the other firm's valuation are equally likely
    - firm *A* believes that V_B is uniformly distributed over the interval [0,1] and vice versa

An Oil Tract Auction

- Each firm must now decide its bid (b_A and b_B)
- The gain from the auction for firm *A* is

$$V_A - b_A \text{ if } b_A > b_B$$

  and

$$0 \text{ if } b_A < b_B$$

- Assume that each player opts to bid a fraction (k_i) of the valuation

An Oil Tract Auction

- Firm *A*'s expected gain from the sale is

$$\pi_A = (V_A - b_A) \cdot \text{Prob}(b_A > b_B)$$

- Since *A* believes that V_B is distributed normally,

$$\text{prob}(b_A > b_B) = \text{prob}(b_A > k_B V_B)$$
$$= \text{prob}(b_A / k_B > V_B) = b_A / k_B$$

- Therefore,

$$\pi_A = (V_A - b_A) \cdot (b_A / k_B)$$

An Oil Tract Auction

- Note that π_A is maximized when

$$b_A = V_A/2$$

- Similarly,

$$b_B = V_B/2$$

- The firm with the highest valuation will win the bid and pay a price that is only 50 percent of the valuation

An Oil Tract Auction

- The presence of additional bidders improves the situation for the seller
- If firm *A* continues to believe that each of its rivals' valutions are uniformly distributed over the [0,1] interval,

$$\text{prob}(b_A > b_i) = \text{prob}(b_A > k_i V_i) \text{ for } i = 1,\ldots,n$$

$$= \prod_{i=1}^{n-1} (b_A / k_i) = b_A^{n-1} / k^{n-1}$$

An Oil Tract Auction

- This means that

$$\pi_A = (V_A - b_A)(b_A^{n-1}/k^{n-1})$$

  and the first-order condition for a maximum is

$$b_A = [(n-1)/n]V_A$$

- As the number of bidders rises, there are increasing incentives for a truthful revelation of each firm's valuation

Dynamic Games with Incomplete Information

- In multiperiod and repeated games, it is necessary for players to update beliefs by incorporating new information provided by each round of play
- Each player is aware that his opponent will be doing such updating
  - must take this into account when deciding on a strategy

Important Points to Note:

- All games are characterized by similar structures involving players, strategies available, and payoffs obtained through their play
  – the Nash equilibrium concept provides an attractive solution to a game
    - each player's strategy choice is optimal given the choices made by the other players
    - not all games have unique Nash equilibria

Important Points to Note:

- Two-person noncooperative games with continuous strategy sets will usually possess Nash equilibria
  - games with finite strategy sets will also have Nash equilibria in mixed strategies

Important Points to Note:

- In repeated games, Nash equilibria that involve only credible threats are called subgame-perfect equilibria

Important Points to Note:

- In a simple single-period game, the Nash-Bertrand equilibrium implies competitive pricing with price equal to marginal cost
- The Cournot equilibrium (with $p > mc$) can be interpreted as a two-stage game in which firms first select a capacity constraint

Important Points to Note:

- Tacit collusion is a possible subgame-perfect equilibrium in an infinitely repeated game
  - the likelihood of such equilibrium collusion diminishes with larger numbers of firms, because the incentive to chisel on price increases

Important Points to Note:

- Some games offer first-mover advantages
  - in cases involving increasing returns to scale, such advantages may result in the deterrence of all entry

Important Points to Note:

- Games of incomplete information arise when players do not know their opponents' payoff functions and must make some conjectures about them
  - in such Bayesian games, equilibrium concepts involve straightforward generalizations of the Nash and subgame- perfect notions encountered in games of complete information