Game Theory and Economics of Contracts
Lecture 6
Extensions of Static Moral Hazard Model

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Existence of Optimal Contracts

- Validity of FOA

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- For abstract mathematical setting,
  Page, (1987), *The Existence of Optimal Contracts in the Principal-Agent Model,* Journal of Mathematical Economics 16, 157-167."
  Kadan,O, Reny, P.J. and Swinkels, J.M., (2011), *Existence of Optimal Contracts in Principal-Agent Problems.* Working Paper
  Chen and Page, (2013) Optimal Contracts with the Inclusion of Observable Actions in Moral Hazard Problems

P-A Problem with Bargaining

- P solves the following program

$$\max_{w \in \mathcal{F}; a \in \mathcal{A}} \int_{\mathcal{X}} [V(x - w(x))]^{\alpha}$$
$$[u(w(x))P(dx|a) - c(a)]^{(1-\alpha)} P(dx|a)$$

$$s.t. \int_{\mathcal{X}} u(w(x))P(dx|a) - c(a) \geq r, (IR)$$

$$a \in \arg\max_{a' \in \mathcal{A}} \int_{\mathcal{X}} u(w(x))P(dx|a') - c(a'), (IC)$$

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- $\alpha>0$ denotes the weight of bargaining power of P. $1-\alpha$ denotes the weight of bargaining power of A. Recall Nash Bargaining. Once $\alpha=1$, P has full bargaining power.

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  2. In technology licensing, the services of the licensor's personnel to install and train the licensee's personnel and other relevant services.
  3. In warranty contracts, the buyer (A) takes hidden action which influences the performance of the product. The seller (P) pick a quality level for the product. The warranty contract will be contingent on product performance.

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- Game tree.

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- Theorem (Bhattacharyya and Lafontaine 1995 RJE)
  If $X = f(a_1, a_2) + \varepsilon$,where X is the total monetary return produced (random variable of x) and ε is a random term with mean zero and variance σ^2. P and A are both risk neutral. Without loss of generality, the optimal sharing rule (contract) in double moral hazard model can be represented by a linear contract.

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- With double-sided moral hazard the optimal contract $\alpha x + \beta$ cannot have $\beta = 0$ or $\beta = 1$. In other words, output must be shared.

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- In many realistic cases, the principal delegates a task to many (a team of) agents. Those agents' actions will be interdependently influence the outcome of the delegation.
- We have a **partnership** when there is effectively no principal and so the agents split the output amongst themselves; i.e., the wage schedule satisfies *budget balance*: An important aspect of a partnership is that the budget (i.e., transfers) are always exactly balanced on and off the equilibrium path.

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- A contract is a collection of wage schedules $s = (s_1, \cdots, s_N)$, where each agent's wage schedule indicates the transfer the agent receives as a function of verifiable output; i.e., $s_i : \mathcal{X} \to \mathbb{R}$.

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  where m_i denotes monetary income, and $C_i(a_i)$ is cost function that is strictly increasing and convex. (Note that agents are risk neutral in money.) Let r_i be outside option (which determines each agent's IR).

Canonical multi-agent model

- The timing of the game is as follows.
  At stage 1, the principal offers a wage schedule s for each agent which is observable by everyone.
  At stage 2, the agents reject the contract or accept and simultaneously and noncooperatively select a_i.
  At stage 3, the output level x is realized and participating agents are paid appropriately.

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  At stage 3, the output level x is realized and participating agents are paid appropriately.
- In this case, we say we have a partnership when there is effectively no principal and so the agents split the output amongst themselves $\sum s_i(x) = x, \forall x \in X$.

First Best: The Planner's Problem

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- Assume that there are no incentive problems, and that a planner can enforce agents to choose a particular action (so we are also ignoring IR constraints.)
- The first best solution maximizes total surplus, and solves,

$$\max_a x(a) - \sum_i C_i(a_i)$$

which yields the FOCs,

$$\frac{\partial x(a)}{\partial a_i} = C_i'(a_i) \ \forall i = 1, ..., N.$$

That is, the marginal benefit from agent i's action equals the marginal private cost of agent i.

Second Best: The "Partnership" Problem

- Consider the partnership problem where the agents jointly own the output. Assume that the actions are not contractible and the agents must resort to an incentive scheme $\{s_i(x)\}_{i=1}^n$ under the restriction of a balanced budget ("split-the-pie") rule: $\sum_i s_i(x) = x, \forall x$.

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- We also impose a "limited liability" restriction, $s_i(x) \geq 0, \forall x$.
- We solve the partnership problem using Nash Equilibrium (NE), and it is easy to see that given an incentive scheme $\{s_i(x)\}_{i=1}^n$, any NE must satisfy

$$s_i'(x)\frac{\partial x}{\partial a_i} = C_i'(a_i)$$

Why?

Can the Partnership achieve FB.?

- For the partnership to achieve FB efficiency, we must find an incentive scheme for which the NE coincides with the FB solution. That is, from the FOC of the FB problem and NE conditions, we must have $s_i'(x) \equiv 1, \forall x$, and $\forall i$,which implies that $\sum_i s_i'(x) = n$.

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- But from budget balance condition we have $\sum_i s_i'(x) = 1, \forall x$,which implies that a budget balanced partnership cannot achieve FB efficiency.
- The intuition is standard, and is related to the "Free riding" problem common to such problems of externalities.

A solution: Budget-Breaker Principal

- We now introduce a new, (n+1)th player to the partnership who will play the role of the "Budget-Breaker." This is just another case of an "unproductive" principal who helps to solve the partnership problem.

$$s_i(x) = \begin{cases} s_i^* & if\ x \geq x_{FB}^* \\ 0 & if\ x < x_{FB}^* \end{cases}, i = 1, \cdots, n$$

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- where s_i^* is arbitrarily chosen to satisfy: $\sum_{i=1}^{N} s_i^* = x_{FB}^*$,and $s_i^* > C_i(a_i^*)$, $\forall i$.
- In equilibrium, $\sum_{i=1}^{N+1} s_i(x) = x_{FB}^*$, and $s_{n+1}(x_{FB}^*) = 0$.

Relative Performance Evaluations

Now we change the setting by adding uncertainty and multidimensional outputs.

- n risk averse agents, each with utility over income/action as before, $u_i(m_i, a_i) = u_i(m_i) - C_i(a_i)$, with $u' > 0, u'' < 0, C' > 0$, and $C'' > 0$.

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- $y = (x, y_1, y_2, ..., y_m)$ is a vector of random variables (e.g., outputs, or other signals) which is dependent on the vector of actions, $a = (a_1, a_2, ..., a_n)$. Let project function $\pi(y) = x$ (the profit).

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- $P(y|a)$ denotes the probability measure of principal's expected profit conditional on the actions. It can have a density $g(y|a)$.

Relative Performance Evaluations

Now we have the principal's problem

$$\max_{a,s(y)} \int_y [\pi(y) - \sum_i s_i(y)] P(y|a)$$

$$s.t. \int_y u_i[s_i(y)] P(y|a) - C_i(a_i) \geq r_i, \forall i$$

$$a_i \in \arg\max_{a_i' \in \mathcal{A}_i} \int_y u_i[s_i(y)] P(y|a_i', a_{-i}) - C_i(a_i'), \forall i$$

Subgame perfect equilibrium in the sequential game with the agents' simultaneous move in second stage.
Game tree.

Relative Performance Evaluations

- A function $T_i(y)$ is a **sufficient statistic** for y with respect to a_i if there exist functions $h_i(\cdot|\cdot) \geq 0$ and $p_i(\cdot|\cdot) \geq 0$ such that, $g(y|a) = h_i(y|a-i)p_i(T_i(y)|a), \forall (y, a) \in support(g)$

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- Theorem (Holmstrom, 1982) Assume $T(y) = (T_1(y), ..., T_n(y))$ is sufficient for y with respect to a. Then, given any collection of incentive schemes $\{s_i(y)\}_{i=1}^n$, there exists a set of schemes $\{\tilde{s}_i(T_i(y)\}_{i=1}^n$, that weakly Pareto dominates $\{s_i(y)\}_{i=1}^n$.

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- The intuition is the same as for single agent in Holmstrom (1979): The collection $(T_1, ..., T_n)$ gives better information than y. Thus we can think of y as a garbling of the vector $T(y)$. In that case, Relative performance evaluations are desirable.

Hierarchical MH Model

- Suppose that a principal is interested in hiring two agents, labelled 1 and 2, to carry out two tasks. The monetary outcome of the agents' effort is a random variable x. For simplicity, we assume that x can take N possible values, $X = \{x_1, ..., x_N\}$. (Finite case)

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- Let $P_k(e_1, e_2)$ be the probability of observing the outcome x_k conditioned on the effort levels $(e_1, e_2) \in E_1 \times E_2$, where E_i is the (possibly multidimensional) space of efforts for agent i. Each agent's effort is only observable to himself.

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- A contract for agent i, $i = 1; 2$, is a payment schedule $w^i(x) = (w^i_1(x), ..., w^i_N(x))$.

Hierarchical MH Model

- Agent i's utility function is denoted by $U_i(w^i, e_i)$ and it is assumed to be increasing in the first argument and decreasing in the second. The agent's expected utility is given by

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- Agent i accepts a contract if the expected utility it yields is larger than his reservation utility r_i.
- The principal's utility function is denoted by $V(x, w)$.This function is increasing in the result and decreasing in the agents' reward. The principal's expected utility is given by

$$EV(w^i; e_i; e_j) = \sum_k P_k(e_1, e_2) U_i(x_k, w_k)$$

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- A **centralized** structure in this game is an organization in which Agents 1 and 2 are hired directly by the principal.
- In a **decentralized** structure the principal hires one agent and lets him contract separately with the other agent.
  Two-tier contracting structure!

Centralized Structure

- Principal's problem **P(C)**

$$
\begin{aligned}
& \max_{e,w,} EV(x, w^1 + w^2, e^1; e^2) \\
s.t. EU_i(w^i, e_i, e_j) &\geq r_i, \forall i, j, i \neq j \\
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- Principal's problem **P(C)**

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- A **coalition** is a partnership of two participants, based on a side (cover) contract, signed after the main contracts but before the actions are taken. This contract specifies monetary transfers as a function of the verifiable outcome x.

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- A coalition is formed by having an agent propose a side contract to the other agent, or a third party offering the contract to the agents.
- Now let $z(x) = (z_1(x), ..., z_N(x))$ denote the contingent payment that Agent 1 receives from the principal and $w^2(x) = (w_1^2(x), ..., w_N^2(x))$ denote the one Agent 2 receives from Agent 1.

- Agent 1 become the intermediate agent with a problem **P(z)**

$$\begin{aligned} & \max_{e_1,e_2;w2} EU_1(z-w^2,e_1,e_2) \\ s.t.\ EU^2(w^2,e^2,e^1) &\geq r_2 \\ e_1 &\in \arg\max_{e_1'} EU_1(w^2,e_1',e_2) \\ e_2 &\in \arg\max_{e_2'} EU_1(w^2,e_1,e_2') \end{aligned}$$

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- Given the solution of the game between the two agents (by backward induction), the principal solves program **P(D)**

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s.t. EU_1(w^2; e_1; e_2) \quad & \geq \quad r_1 \\
& (e_1, e_2, w^2) \text{ solves the program } \mathbf{P(z)}
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- Game tree

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- Theorem (Macho-Stadler and Perez-Castrillo 1998) If The utility functions are such that the Participation constraints of both agents are binding, in a moral hazard framework the following two situations yield the same set of equilibria:
  (i) a decentralized structure,
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- Corollary (Macho-Stadler and Perez-Castrillo 1998)
  1) centralized organizations where participants cannot collude are at least as efficient as decentralized ones.
  2) If the principal cannot avoid agents' side-contracting, then the outcome is the same should the principal delegate Agent 2's contract or not.