Game Theory and Economics of Contracts
Lecture 7
Extensions of Moral Hazard Model

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Efficiency in Multi-Agency Model

We consider a situation in which there is a social planner, as the principal, design a contract plan for all the agents to maximize the social welfare.

$$\max_{a,s(y)} \int_Y \sum_i [u_i(s_i(y)) - C_i(a_i)] P(y|a)$$

$$s.t. \int_y u_i(s_i(y)) P(y|a) - C_i(a_i) \geq r_i, \forall i$$

$$a_i \in \arg\max_{a_i' \in \mathcal{A}_i} \int_y u_i[s_i(y)] P(y|a_i', a_{-i}) - C_i(a_i'), \forall i$$

We can add more terms to reflect spillover effect or other surplus things. This model setting is frequently used in regulation contexts.

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- P pays A a wage, $w_t(x^t) \in \mathbb{R}$, based upon **the history of outputs**, $x^t = \{x_1, \cdots, x_t\}$. So it is generally a long-term contract. The outcome (output) in period t is $x_t \in \mathcal{X}_t$. Assume $\mathcal{X}_t \equiv [\underline{x}, \overline{x}]$.

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- $u(w_t)$ is the benefit utility of A from receiving w_t and $C(a_t)$ denotes the cost function of A's taking action a_t. Reservation utility of A is normalized to be 0.
- $f(x_t|a_t)$ denotes the probability density (or point mass) of $x_t's$ occurring during a period t in which action a_t was taken.

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- Under both models, Radner (1985) and Holmstrom and Milgrom (1990) have shown that repeated moral hazard relationships will achieve the first-best arbitrarily close as either $T \to \infty$ (in the time-averaging case) or $\delta \to 1$ (in the discounting case).

Full-blown T-period Dynamic MH Model

$$\max_{\{w_t(\cdot),a_t\}_t} \sum_{t=1}^{T} \delta^{t-1}\{(\prod_{h=1}^{t}\int_{\underline{x}}^{\overline{x}})(x_t - w_t(\prod_{h=1}^{t} x_h)) \prod_{h=1}^{t}(f(x_h|a_h)) \prod_{h=1}^{t} d(x_h)\}$$

s. to **(IC for A in each period)**

$$a_t \in \arg\max_{a'_t}\{(\prod_{h=1}^{t}\int_{\underline{x}}^{\overline{x}})[\textstyle\sum_{h=1}^{t} \delta^{h-1}(u(w_h(\prod_{i=1}^{h} x_i))]$$
$$[\prod_{h=1}^{t-1} f(x_h|a_h)f(x_t|a'_t)] \prod_{h=1}^{t} d(x_h)\} - C(a'_t), \forall t;$$

(IR for A)

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- $\max_{\{w_t(\cdot), a_t\}_t} \int_{\underline{x}}^{\overline{x}} (x_1 - w_1(x_1)) f(x_1|a_1) dx_1 + \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{\overline{x}} (x_2 - w_2(x_1, x_2)) f(x_1|a_1) f(x_2|a_2) dx_1 dx_2\}$

  s.to (FOC)
  $\int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{\overline{x}} [u(w_1(x_1)) + \delta u(w_2(x_1, x_2))] f_a(x_1|a_1) f(x_2|a_2) dx_1 dx_2 = C'(a_1)$,

  $\int_{\underline{x}}^{\overline{x}} [u(w_2(x_1, x_2))] f_a(x_2|a_2) dx_2 = C'(a_2)$,

  $\int_{\underline{x}}^{\overline{x}} [u(w_1(x_1))] f(x_1|a_1) dx_2 - C(a_2) + \int_{\underline{x}}^{\overline{x}} \int_{\underline{x}}^{\overline{x}} \delta u(w_2(x_1, x_2))] f(x_1|a_1) f(x_2|a_2) dx_1 dx_2 - C(a_2) \geq 0$,

Two period Dynamic MH Model

- Let μ_1, $\mu_2(x_1)$ and λ represent the multipliers associated with each constraint (note that the second constraint - the IC constraint for period 2 - depends upon x_1, and so there is a separate constraint and multiplier for each x_1).

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- Let μ_1, $\mu_2(x_1)$ and λ represent the multipliers associated with each constraint (note that the second constraint - the IC constraint for period 2 - depends upon x_1, and so there is a separate constraint and multiplier for each x_1).
- After Differentiating and simplifying, one obtains:
  $\frac{1}{u'(w_1(x_1))} = \lambda + \mu_1 \frac{f_a(x_1,a_1)}{f(x_1,a_1)}, \forall x_1$
  $\frac{1}{u'(w_2(x_1,x_2))} = \lambda + \mu_1 \frac{f_a(x_1,a_1)}{f(x_1,a_1)} + \mu_2(x_1) \frac{f_a(x_2,a_2)}{f(x_2,a_2)}, \forall x_1, x_2$

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- Combine them and we have

$$\frac{1}{u'(w_2(x_1, x_2))} = \frac{1}{u'(w_1(x_1))} + \mu_2(x_1)\frac{f_a(x_2, a_2)}{f(x_2, a_2)}$$

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- It is **Borch-Like rule**. The difference of marginal rates of substitution between P and A w.r.t monetary income across time is equal to a deviation as $\mu_2(x_1)\frac{f_a(x_2,a_2)}{f(x_2,a_2)}$.

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- People also have interest in comparison between *long term and short term contracts*. Rogerson (1985) shows that in a two-period repeated MH model with finitely many states, if the players have no access to capital market, i.e. $\delta \equiv 1$, and $w_1(x_i) \neq w_1(x_j), \forall x_i, x_j \in \mathcal{X}_1$, then the contract must be long term, i.e., there exists a $x_k \in \mathcal{X}_2$ such that

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- In continuum cases, there could be more variations.

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- Engineering in economics!

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- One must use abstraction in theoretical modelling to capture the most significant/interesting part of real life.
- The elegance of economics modeling is embodied in the best balance of abstraction, clarity, and its power of interpretation.
- But theory is still of importance, because it provides the benchmark or tool for the analysis and application. Game theory is the foundation for contract theory, contract theory is the foundation for many applied fields, e.g. executive compensation, insurance, regulation, etc.