Games and Contracts
Lecture 9
Basics of Adverse Selection Model

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Fall 2015

Adverse Selection: Discrete Types

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- Then, the principal's problem is:

$$
\begin{aligned}
& \max_{\{(t_i,x_i)\}_{i=1}^n} \sum_{i=1}^n \pi_i(t_i - c(x_i)) \\
s.t. v(x_i, \theta_i) - t_i &\geq 0, \forall i \quad (IR) \\
v(x_i, \theta_i) - t_i &\geq v(x_j, \theta_i) - t_j, \forall i \neq j \ (IC),
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- This is, however, a complicated problem, especially as n grows large: There are a total of n (IR) constraints and another $n(n-1)$ (IC) constraints.

Adverse Selection: Discrete Types

- Proposition 3.1: (Maskin-Riley) The principal's problem reduces to:

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$$\begin{aligned} s.t. v(x_1,\theta_1) - t_1 &\geq 0 && (IR_1) \\ v(x_i,\theta_i) - t_i &\geq v(x_{i-1},\theta_i) - t_{i-1}, \forall i = 2,\cdots,n && (DIC), \\ x_i &\geq x_{i-1}, \forall i = 2,\cdots,n && (MON) \end{aligned}$$

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- That is, there is one (IR) constraint, $(n-1)$ "Downward" (IC) constraints, and another $(n-1)$ "Monotonicity" constraints. These features are features of the solution, that must hold for any solution under the assumptions that we usually make.

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  $v(\mathbf{x}(\theta), \theta) - \mathbf{t}(\theta) \geq v(\mathbf{x}(\theta'), \theta) - \mathbf{t}(\theta'), \forall \theta, \theta' \in \Theta \quad (IC)$

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- Proposition Suppose IC mechanism $(\mathbf{x}, \mathbf{t})$ is differentiable. $(\mathbf{x}, \mathbf{t})$ is incentive compatible if and only if
  (M) $\mathbf{x}'(\theta) \geq 0$, $\forall \theta$.
  (ICFOC) $v_x(\mathbf{x}(\theta), \theta)\mathbf{x}'(\theta) - \mathbf{t}'(\theta) = 0$, $\forall \theta$,

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- Given SCP of v, at a solution $(\mathbf{t}, \mathbf{x})$, $\text{IR}_{\underline{\theta}}$ is binding, i.e. $v(\mathbf{x}(\underline{\theta}), \underline{\theta}) - \mathbf{t}(\underline{\theta}) = v(0, \underline{\theta})$.

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- From the analysis above we can rewrite the principal's problem as
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- To solve this program in general requires optimal control theory, but this can sometimes be avoided by the following Shortcut: We solve the relaxed program obtained by ignoring the monotonicity constraint (M). If it turns out that the resulting solution satisfies (M), then we are done.

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- Define the Agent's equilibrium utility as $U(\theta) \equiv v(\mathbf{x}(\theta), \theta) - \mathbf{t}(\theta)$, which depends on θ in two ways – through the agent's true type and through his truthful announcement.

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- To solve the relaxed problem, ICFOC can equivalently be written as

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} v_\theta(\mathbf{x}(s), s)ds$$

and the binding (IR) means

$$U(\underline{\theta}) = v(0, \underline{\theta}),$$

thus (ICFOC) and (IR) together are equivalent to

$$U(\theta) = v(0, \theta) + \int_{\underline{\theta}}^{\overline{\theta}} v_\theta(\mathbf{x}(s), s)ds.$$

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- Eliminating the constant term $v(0, \theta)$, the objective function takes the familiar form as the expected difference between **total surplus** and the Agent's **information rent** (extra cost caused by hidden information):

$$\max_{\mathbf{x}} \int_{\underline{\theta}}^{\overline{\theta}} [v(\mathbf{x}(\theta), \theta) - c(\mathbf{x}(\theta))] f(\theta) d\theta - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\mathbf{x}(s), s) ds f(\theta) d\theta.$$

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- We can rewrite the expected information rents using integration by parts:

$$\begin{aligned}
& \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\mathbf{x}(s), s) ds f(\theta) d\theta \\
= & \int_{\underline{\theta}}^{\theta} v_\theta(\mathbf{x}(s), s) ds F(\theta) \Big|_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} v_\theta(\mathbf{x}(\theta), \theta) F(\theta) d\theta \\
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- With the expected information rents given above, we can rewrite the principal's problem as:

$$\max_{\mathbf{x}} \int_{\underline{\theta}}^{\overline{\theta}} [v(\mathbf{x}(\theta), \theta) - c(\mathbf{x}(\theta)) - v_\theta(\mathbf{x}(\theta), \theta) \frac{1 - F(\theta)}{f(\theta)}] f(\theta) d\theta.$$

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- Thus, P will maximize the expected value of the expression within square brackets, which is called the **virtual surplus**.

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- Any pointwise maximizer,

$$\mathbf{x}^*(\theta) \in \arg\max_x \int_{\underline{\theta}}^{\overline{\theta}} [v(x,\theta) - c(x) - v_\theta(x,\theta)\frac{1-F(\theta)}{f(\theta)}] f(\theta) d\theta,$$

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- **Proposition** If v has SCP ($v_{x\theta} > 0$), $v_{x\theta\theta} \leq 0$, and $\frac{d(\frac{f(\theta)}{1-F(\theta)})}{d\theta} \geq 0$, then $\mathbf{x}^*$ is increasing and therefore solution to relaxed problem satisfies (M) and solve the full problem.

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- The government (regulator) maximizes social welfare:

$$B(y) - (1 - \lambda)s + s - \psi(y, \theta),$$

where $B(y) = \int_0^y p(y)dy$ is the social surplus from producing y, and $\lambda > 0$ is the "shadow cost" of distortionary taxes (taxes are needed to collect the subsidy s). (Everything is common knowledge except θ.)

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- The government can offer the firm a mechanism or menu: $(\mathbf{y}(\theta), \mathbf{s}(\theta))$ and the firm's profits are,

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- (That is, we can redefine the subsidy s to include the revenues that the government can collect and transfer to the firm.) The government must assure that $u(\mathbf{y}, \mathbf{s}, \theta) \geq 0$, which is the IR constraint, and must also respect the IC constraints of truthful revelation.

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- Letting $c(\mathbf{y}(\theta)) \equiv \psi(\mathbf{y}(\theta), \theta) - B(\mathbf{y}(\theta))$, the government maximizes:

$$\max_{\mathbf{x};\mathbf{t}} \int_{\underline{\theta}}^{\overline{\theta}} [\lambda t(\theta) - c(x(\theta))] f(\theta) d\theta$$

subject to the standard IR and IC.

Applications: Optimal Labor Contracts

- Consider the case where the manager-owner of a firm is risk neutral and the employee is risk averse to the amount of labor input. That is, assume that the worker's utility is given by

  $$u(\ell, w, \theta) = w - \psi(\ell, \theta),$$

  and the owner's utility is given by,

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  where θ is that marginal product of the worker, w is the wage the worker receives, and ℓ is the worker's labor input.
- ℓ, θ are assumed to be private information of the worker, and the employer only observes the output $\theta\ell$. We can now redefine the variables so as to put this problem in the notation of our original model. That is, let

  $$y(\ell, \theta) \equiv -\ell\theta; \mathbf{t}(\theta) \equiv -w(\theta); v(x, \theta) \equiv -\psi(\ell, \theta) = -\psi(-\frac{x}{\theta}, \theta),$$

  which yields the exact same problem.

Applications: Vertical Differentiation-Quality

- A monopoly manufactures goods in 1-unit quantities each, but they can differ in quality. Just take x to be quality of a unit of good, and $c(x)$ to be the cost of producing one unit at quality x, and we are back in the model we analyzed.

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- This model was analyzed by Mussa-Rosen (1978) and they note the connection to Mirrlees' work but just apply it to this problem. (Examples: train/plane classes, "olives" and "figs" restaurants in Charlestown, 486SX and 486DX computer chips.)

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- Each agent i observes a private signal which determines his preferences over alternatives $x \in X$, the signal for each $i : \theta_i \in \Theta_i$.
- The principal has the ex post utility function $v(x, \theta)$.

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- The vector of types, $\theta = (\theta_1, ..., \theta_n) \in \Theta_1 \times ... \times \Theta_n \equiv \Theta$ is drawn from a prior distribution with density $p(\theta)$ [can be probabilities for finite Θ]. θ is also called **the state of the world**.

Multi-Agency Adverse Selection

- Let $p_{-i}(\theta_{-i}|\theta_i)$ be i's probability assessment over the possible types of other agents given his type is θ_i. $p_{-i}(\theta_{-i}|\theta_i)$ denotes i's marginal probability assessment over his own types.

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- Information:
  (1) θ_i is privately observed by agent i
  (2) $\{u_i(\cdot)\}_i$ is common knowledge
  (3) p (or also p_{-i}) is common knowledge

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- Posed as a mechanism design question, the principal will want to choose the game (from the set of all possible games) which has the best equilibrium (to be defined) for the principal.
- A communication mechanism $\mathbf{y} : \prod_i M_i \to Y$ is a function associating a joint message (i.e., strategy) space for each agent, M_i, with an allocation y. Let $m = (m_1, \cdots, m_n) \in M = (M_1, \cdots, M_n)$. For generality, we will suppose that M_i includes all possible mixtures over messages; thus, m_i may be a probability distribution.
- In essence, any mechanism defines a simultaneous-move subgame for the agents to play (to report some messages).

Multi-Agency Adverse Selection

The timing of the communication mechanism game is as follows:

1. The principal offers a communication mechanism $\mathbf{y}(m)$ to the agents.
2. The agents simultaneously decide whether or not to participate in the mechanism. (This stage may be superfluous in some contexts; moreover, we can always require the principal include the message of "I do not wish to play" and the null contract, making the acceptance stage unnecessary.)
3. Agents play the communication mechanism by sending messages to the principal simultaneously.

Game tree: we must choose an equilibrium concept for subgame played by the agents. We may consider BNE (or Strategy-proof equilibria, etc.).