Chapter 15

GAME THEORY MODELS OF PRICING
Game Theory

- Game theory involves the study of strategic situations
- Game theory models attempt to portray complex strategic situations in a highly simplified and stylized setting
  - abstract from personal and institutional details in order to arrive at a representation of the situation that is mathematically tractable
Game Theory

• All games have three elements
  – players
  – strategies
  – payoffs

• Games may be cooperative or noncooperative
Players

• Each decision-maker in a game is called a **player**
  – can be an individual, a firm, an entire nation
• Each player has the ability to choose among a set of possible actions
• The specific identity of the players is irrelevant
  – no “good guys” or “bad guys”
Strategies

• Each course of action open to a player is called a strategy
• Strategies can be very simple or very complex
  – each is assumed to be well-defined
• In noncooperative games, players are uncertain about the strategies used by other players
Payoffs

• The final returns to the players at the end of the game are called payoffs.
• Payoffs are usually measured in terms of utility.
  – Monetary payoffs are also used.
• It is assumed that players can rank the payoffs associated with a game.
Notation

- We will denote a game $G$ between two players ($A$ and $B$) by

$$G[S_A, S_B, U_A(a,b), U_B(a,b)]$$

where

- $S_A = \text{strategies available for player } A \ (a \subset S_A)$
- $S_B = \text{strategies available for player } B \ (b \subset S_B)$
- $U_A = \text{utility obtained by player } A \ \text{when particular strategies are chosen}$
- $U_B = \text{utility obtained by player } B \ \text{when particular strategies are chosen}$
Nash Equilibrium in Games

• At market equilibrium, no participant has an incentive to change his behavior.

• In games, a pair of strategies \((a^*, b^*)\) is defined to be a Nash equilibrium if \(a^*\) is player \(A\)'s best strategy when player \(B\) plays \(b^*\), and \(b^*\) is player \(B\)'s best strategy when player \(A\) plays \(a^*\).
Nash Equilibrium in Games

• A pair of strategies \((a^*, b^*)\) is defined to be a Nash equilibrium if

\[
U_A(a^*, b^*) \geq U_A(a', b^*) \quad \text{for all } a' \subset S_A
\]

\[
U_B(a^*, b^*) \geq U_B(a^*, b') \quad \text{for all } b' \subset S_B
\]
Nash Equilibrium in Games

• If one of the players reveals the equilibrium strategy he will use, the other player cannot benefit
  – this is not the case with nonequilibrium strategies

• Not every game has a Nash equilibrium pair of strategies

• Some games may have multiple equilibria
A Dormitory Game

• Suppose that there are two students who must decide how loudly to play their stereos in a dorm
  – each may choose to play it loudly (L) or softly (S)
A Dormitory Game

A chooses loud (L) or soft (S)

B makes a similar choice

Neither player knows the other’s strategy

Payoffs are in terms of A’s utility level and B’s utility level.
A Dormitory Game

• Sometimes it is more convenient to describe games in tabular ("normal") form.

<table>
<thead>
<tr>
<th>A’s Strategies</th>
<th>B’s Strategies</th>
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<tbody>
<tr>
<td>L</td>
<td>7,5 5,4</td>
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<tr>
<td>S</td>
<td>6,4 6,3</td>
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</tbody>
</table>

A’s Strategies:

- Sometimes it is more convenient to describe games in tabular ("normal") form.
A Dormitory Game

• A loud-play strategy is a dominant strategy for player $B$
  – the $L$ strategy provides greater utility to $B$ than does the $S$ strategy no matter what strategy $A$ chooses

• Player $A$ will recognize that $B$ has such a dominant strategy
  – $A$ will choose the strategy that does the best against $B$’s choice of $L$
A Dormitory Game

- This means that A will also choose to play music loudly
- The $A:L, B:L$ strategy choice obeys the criterion for a Nash equilibrium
  - because $L$ is a dominant strategy for $B$, it is the best choice no matter what $A$ does
  - if $A$ knows that $B$ will follow his best strategy, then $L$ is the best choice for $A$
Existence of Nash Equilibria

• A Nash equilibrium is not always present in two-person games
• This means that one must explore the details of each game situation to determine whether such an equilibrium (or multiple equilibria) exists
No Nash Equilibria

- Any strategy is unstable because it offers the other players an incentive to adopt another strategy.

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<tr>
<th>A’s Strategies</th>
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<td>Rock</td>
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<td>Rock</td>
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<tr>
<td>Paper</td>
<td>-1,1</td>
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<td>Scissors</td>
<td>1,-1</td>
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Two Nash Equilibria

- Both of the joint vacations represent Nash equilibria

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<thead>
<tr>
<th>A’s Strategies</th>
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<tr>
<td>Mountain</td>
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<td>1,2</td>
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Existence of Nash Equilibria

• There are certain types of two-person games in which a Nash equilibrium must exist
  – games in which the participants have a large number of strategies
    • games in which the strategies chosen by A and B are alternate levels of a single continuous variable
    • games where players use mixed strategies
Existence of Nash Equilibria

- In a game where players are permitted to use mixed strategies, each player may play the pure strategies with certain, pre-selected probabilities
  - player A may flip a coin to determine whether to play music loudly or softly
  - the possibility of playing the pure strategies with any probabilities a player may choose, converts the game into one with an infinite number of mixed strategies
The Prisoners’ Dilemma

- The most famous two-person game with an undesirable Nash equilibrium outcome

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<thead>
<tr>
<th>A’s Strategies</th>
<th>B’s Strategies</th>
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<tbody>
<tr>
<td>Confess</td>
<td>Confess</td>
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<tr>
<td>A: 3 years</td>
<td>A: 6 months</td>
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<tr>
<td>B: 3 years</td>
<td>B: 10 years</td>
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<td>Confess</td>
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<td>A: 10 years</td>
<td>A: 2 years</td>
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<tr>
<td>B: 6 months</td>
<td>B: 2 years</td>
</tr>
<tr>
<td>Not Confess</td>
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</tbody>
</table>
The Prisoners’ Dilemma

• An ironclad agreement by both prisoners not to confess will give them the lowest amount of joint jail time
  – this solution is not stable

• The “confess” strategy dominates for both $A$ and $B$
  – these strategies constitute a Nash equilibrium
The Tragedy of the Common

• This example is used to signify the environmental problems of overuse that occur when scarce resources are treated as “common property”

• Assume that two herders are deciding how many of their yaks they should let graze on the village common
  – problem: the common is small and can rapidly become overgrazed
The Tragedy of the Common

• Suppose that the per yak value of grazing on the common is

\[ V(Y_A, Y_B) = 200 - (Y_A + Y_B)^2 \]

where \( Y_A \) and \( Y_B \) = number of yaks of each herder

• Note that both \( V_i < 0 \) and \( V_{ii} < 0 \)
  – an extra yak reduces \( V \) and this marginal effect increases with additional grazing
The Tragedy of the Common

• Solving herder A’s value maximization problem:
  \[ \text{Max } Y_A V = \text{Max } [200Y_A - Y_A(Y_A + Y_B)^2] \]

• The first-order condition is
  \[ 200 - 2Y_A^2 - 2Y_AY_B - Y_A^2 - 2Y_AY_B - Y_B^2 = 200 - 3Y_A^2 - 4Y_AY_B - Y_B^2 = 0 \]

• Similarly, for B the optimal strategy is
  \[ 200 - 3Y_B^2 - 4Y_BY_A - Y_A^2 = 0 \]
The Tragedy of the Common

• For a Nash equilibrium, the values for $Y_A$ and $Y_B$ must solve both of these conditions

• Using the symmetry condition $Y_A = Y_B$

\[200 = 8Y_A^2 = 8Y_B^2\]

\[Y_A = Y_B = 5\]

• Each herder will obtain 500 [$= 5 \cdot (200-10^2)$] in return

• Given this choice, neither herder has an incentive to change his behavior
The Tragedy of the Common

- The Nash equilibrium is not the best use of the common
- $Y_A = Y_B = 4$ provides greater return to each herder $[4 \cdot (200 - 8^2) = 544]$  
- But $Y_A = Y_B = 4$ is not a stable equilibrium
  - if $A$ announces that $Y_A = 4$, $B$ will have an incentive to increase $Y_B$
  - there is an incentive to cheat
Cooperation and Repetition

• Cooperation among players can result in outcomes that are preferred to the Nash outcome by both players
  – the cooperative outcome is unstable because it is not a Nash equilibrium

• Repeated play may foster cooperation
A Two-Period Dormitory Game

• Let’s assume that A chooses his decibel level first and then B makes his choice.

• In effect, that means that the game has become a two-period game.
  – B’s strategic choices must take into account the information available at the start of period two.
A Two-Period Dormitory Game

A chooses loud (L) or soft (S)

B makes a similar choice knowing A’s choice

Thus, we should put B’s strategies in a form that takes the information on A’s choice into account
A Two-Period Dormitory Game

- Each strategy is stated as a pair of actions showing what B will do depending on A’s actions

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<tr>
<th>A’s Strategies</th>
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<tr>
<td>L</td>
<td>7,5</td>
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<td>S</td>
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</table>
A Two-Period Dormitory Game

- There are 3 Nash equilibria in this game
  - A:L, B:(L,L)
  - A:L, B:(L,S)
  - A:S, B:(S,L)

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A Two-Period Dormitory Game

- A: L, B: (L, S) and A: S, B: (S, L) are implausible
  - each incorporates a noncredible threat on the part of B

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<th>L,L</th>
<th>L,S</th>
<th>S,L</th>
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<tbody>
<tr>
<td>L</td>
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A Two-Period Dormitory Game

• Thus, the game is reduced to the original payoff matrix where \((L,L)\) is a dominant strategy for \(B\)
  – \(A\) will recognize this and will always choose \(L\)

• This is a **subgame perfect equilibrium**
  – a Nash equilibrium in which the strategy choices of each player do not involve noncredible threats
Subgame Perfect Equilibrium

- A “subgame” is the portion of a larger game that begins at one decision node and includes all future actions stemming from that node.
- To qualify to be a subgame perfect equilibrium, a strategy must be a Nash equilibrium in each subgame of a larger game.
Repeated Games

• Many economic situations can be modeled as games that are played repeatedly
  – consumers’ regular purchases from a particular retailer
  – firms’ day-to-day competition for customers
  – workers’ attempts to outwit their supervisors
Repeated Games

• An important aspect of a repeated game is the expanded strategy sets that become available to the players
  – opens the way for credible threats and subgame perfection
Repeated Games

• The number of repetitions is also important
  – in games with a fixed, finite number of repetitions, there is little room for the development of innovative strategies
  – games that are played an infinite number of times offer a much wider array of options
Prisoners’ Dilemma Finite Game (skipped)

• If the game was played only once, the Nash equilibrium $A:U, B:L$ would be the expected outcome

\[
\begin{array}{c|cc}
& L & R \\
\hline
U & 1,1 & 3,0 \\
D & 0,3 & 2,2 \\
\end{array}
\]
Prisoners’ Dilemma Finite Game

- This outcome is inferior to $A:D$, $B:R$ for each player

<table>
<thead>
<tr>
<th></th>
<th>$B$'s Strategies</th>
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<tr>
<td>$A$’s Strategies</td>
<td>$L$</td>
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<td>$U$</td>
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<tr>
<td>$D$</td>
<td>0,3</td>
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</table>
Prisoners’ Dilemma Finite Game

• Suppose this game is to be repeatedly played for a finite number of periods \((T)\)

• Any expanded strategy in which \(A\) promises to play \(D\) in the final period is not credible
  – when \(T\) arrives, \(A\) will choose strategy \(U\)

• The same logic applies to player \(B\)
Prisoners’ Dilemma Finite Game

• Any subgame perfect equilibrium for this game can only consist of the Nash equilibrium strategies in the final round
  – A:U, B:L

• The logic that applies to period $T$ also applies to period $T-1$

• The only subgame perfect equilibrium in this finite game is to require the Nash equilibrium in every round
Game with Infinite Repetitions

• In this case, each player can announce a “trigger strategy”
  – promise to play the cooperative strategy as long as the other player does
  – when one player deviates from the pattern, the game reverts to the repeating single-period Nash equilibrium
Game with Infinite Repetitions

• Whether the twin trigger strategy represents a subgame perfect equilibrium depends on whether the promise to play cooperatively is credible

• Suppose that A announces that he will continue to play the trigger strategy by playing cooperatively in period $K$
Game with Infinite Repetitions

• If $B$ decides to play cooperatively, payoffs of 2 can be expected to continue indefinitely

• If $B$ decides to cheat, the payoff in period $K$ will be 3, but will fall to 1 in all future periods
  – the Nash equilibrium will reassert itself
Game with Infinite Repetitions

• If $\delta$ is player $B$’s discount rate, the present value of continued cooperation is

\[2 + \delta^2 + \delta^2 \delta^2 + \ldots = \frac{2}{1-\delta}\]

• The payoff from cheating is

\[3 + \delta + \delta^2 \delta + \ldots = 3 + \frac{1}{1-\delta}\]

• Continued cooperation will be credible if

\[\frac{2}{1-\delta} > 3 + \frac{1}{1-\delta}\]

\[\delta > \frac{1}{2}\]
The Tragedy of the Common Revisited (skipped)

• The overgrazing of yaks on the village common may not persist in an infinitely repeated game

• Assume that each herder has only two strategies available
  – bringing 4 or 5 yaks to the common

• The Nash equilibrium \((A:5, B:5)\) is inferior to the cooperative outcome \((A:4, B:4)\)
The Tragedy of the Common Revisited

- With an infinite number of repetitions, both players would find it attractive to adopt cooperative trigger strategies if

\[
\frac{544}{1-\delta} > 595 + 500(1-\delta)
\]

\[
\delta > \frac{551}{595} = 0.93
\]
Pricing in Static Games

• Suppose there are only two firms (A and B) producing the same good at a constant marginal cost (c)
  – the strategies for each firm consist of choosing prices ($P_A$ and $P_B$) subject only to the condition that the firm’s price must exceed $c$

• Payoffs in the game will be determined by demand conditions
Pricing in Static Games

• Because output is homogeneous and marginal costs are constant, the firm with the lower price will gain the entire market

• If $P_A = P_B$, we will assume that the firms will share the market equally
Pricing in Static Games

• In this model, the only Nash equilibrium is $P_A = P_B = c$
  – if firm $A$ chooses a price greater than $c$, the profit-maximizing response for firm $B$ is to choose a price slightly lower than $P_A$ and corner the entire market
  – but $B$’s price (if it exceeds $c$) cannot be a Nash equilibrium because it provides firm $A$ with incentive for further price cutting
Pricing in Static Games

• Therefore, only by choosing $P_A = P_B = c$ will the two firms have achieved a Nash equilibrium
  – we end up with a competitive solution even though there are only two firms
• This pricing strategy is sometimes referred to as a Bertrand equilibrium
Pricing in Static Games

• The Bertrand result depends crucially on the assumptions underlying the model
  – if firms do not have equal costs or if the goods produced by the two firms are not perfect substitutes, the competitive result no longer holds
Pricing in Static Games

• Other duopoly models that depart from the Bertrand result treat price competition as only the final stage of a two-stage game in which the first stage involves various types of entry or investment considerations for the firms.
Pricing in Static Games

• Consider the case of two owners of natural springs who are deciding how much water to supply

• Assume that each firm must choose a certain capacity output level
  – marginal costs are constant up to that level and infinite thereafter
Pricing in Static Games

• A two-stage game where firms choose capacity first (and then price) is formally identical to the Cournot analysis
  – the quantities chosen in the Cournot equilibrium represent a Nash equilibrium
    • each firm correctly perceives what the other’s output will be
  – once the capacity decisions are made, the only price that can prevail is that for which quantity demanded is equal to total capacity
Pricing in Static Games

• Suppose that capacities are given by $q_A'$ and $q_B'$ and that

$$P' = D^{-1}(q_A' + q_B')$$

where $D^{-1}$ is the inverse demand function

• A situation in which $P_A = P_B < P'$ is not a Nash equilibrium
  – total quantity demanded $> $ total capacity so one firm could increase its profits by raising its price and still sell its capacity
Pricing in Static Games

- Likewise, a situation in which $P_A = P_B > P'$ is not a Nash equilibrium
  - total quantity demanded < total capacity so at least one firm is selling less than its capacity
    - by cutting price, this firm could increase its profits by taking all possible sales up to its capacity
    - the other firm would end up lowering its price as well
Pricing in Static Games

• The only Nash equilibrium that will prevail is $P_A = P_B = P'$
  – this price will fall short of the monopoly price but will exceed marginal cost

• The results of this two-stage game are indistinguishable from the Cournot model
Pricing in Static Games

• The Bertrand model predicts competitive outcomes in a duopoly situation
• The Cournot model predicts monopoly-like inefficiencies
• This suggests that actual behavior in duopoly markets may exhibit a wide variety of outcomes depending on the way in which competition occurs
Repeated Games and Tacit Collusion (skipped)

• Players in infinitely repeated games may be able to adopt subgame-perfect Nash equilibrium strategies that yield better outcomes than simply repeating a less favorable Nash equilibrium indefinitely
  – do the firms in a duopoly have to endure the Bertrand equilibrium forever?
  – can they achieve more profitable outcomes through tacit collusion?
Repeated Games and Tacit Collusion

• With any finite number of replications, the Bertrand result will remain unchanged
  – any strategy in which firm A chooses $P_A > c$ in period $T$ (the final period) offers B the option of choosing $P_A > P_B > c$
  • A’s threat to charge $P_A$ in period $T$ is noncredible
  – a similar argument applies to any period prior to $T$
Repeated Games and Tacit Collusion

• If the pricing game is repeated over infinitely many periods, twin “trigger” strategies become feasible
  – each firm sets its price equal to the monopoly price \( P_M \) providing the other firm did the same in the prior period
  – if the other firm “cheated” in the prior period, the firm will opt for competitive pricing in all future periods
Repeated Games and Tacit Collusion

• Suppose that, after the pricing game has been proceeding for several periods, firm $B$ is considering cheating
  – by choosing $P_B < P_A = P_M$ it can obtain almost all of the single period monopoly profits ($\pi_M$)
Repeated Games and Tacit Collusion

• If firm \( B \) continues to collude tacitly with \( A \), it will earn its share of the profit stream

\[
\frac{\left( \pi_M + \delta \pi_M + \delta^2 \pi_M + \ldots + \delta^n \pi_M + \ldots \right)}{2} = \frac{\pi_M}{2} \left[ \frac{1}{(1-\delta)} \right]
\]

where \( \delta \) is the discount factor applied to future profits
Repeated Games and Tacit Collusion

• Cheating will be unprofitable if
  \[ \pi_M < \left( \frac{\pi_M}{2} \right) \frac{1}{1 - \delta} \]
  or if
  \[ \delta > \frac{1}{2} \]

• Providing that firms are not too impatient, the trigger strategies represent a subgame perfect Nash equilibrium of tacit collusion
Tacit Collusion

• Suppose only two firms produce steel bars for jailhouse windows
• Bars are produced at a constant $AC$ and $MC$ of $10$ and the demand for bars is
  \[ Q = 5,000 - 100P \]
• Under Bertrand competition, each firm will charge a price of $10$ and a total of
  4,000 bars will be sold
Tacit Collusion

• The monopoly price in this market is $30
  – each firm has an incentive to collude
  – total monopoly profits will be $40,000 each period (each firm will receive $20,000)
  – any one firm will consider a next-period price cut only if $40,000 > $20,000 \(1/1-\delta\)
    • \(\delta\) will have to be fairly high for this to occur
Tacit Collusion

• The viability of a trigger price strategy may depend on the number of firms
  – suppose there are 8 producers
  – total monopoly profits will be $40,000 each period (each firm will receive $5,000)
  – any one firm will consider a next-period price cut if $40,000 > $5,000 \(1/1-\delta\)

• this is likely at reasonable levels of \(\delta\)
Generalizations and Limitations

- The viability of tacit collusion in game theory models is very sensitive to the assumptions made.
- We assumed that:
  - Firm B can easily detect that firm A has cheated.
  - Firm B responds to cheating by adopting a harsh response that not only punishes A, but also condemns B to zero profits forever.
Generalizations and Limitations

• In more general models of tacit collusion, these assumptions can be relaxed
  – difficulty in monitoring other firm’s behavior
  – other forms of punishment
  – differentiated products
Entry, Exit, and Strategy

- In previous models, we have assumed that entry and exit are driven by the relationship between the prevailing market price and a firm’s average cost.
- The entry and exit issue can become considerably more complex.
Entry, Exit, and Strategy

A firm wishing to enter or exit a market must make some conjecture about how its actions will affect the future market price.

- this requires the firm to consider what its rivals will do.
- this may involve a number of strategic ploys.
  - especially when a firm’s information about its rivals is imperfect.
Sunk Costs and Commitment

• Many game theoretic models of entry stress the importance of a firm’s commitment to a specific market
  – large capital investments that cannot be shifted to another market will lead to a large level of commitment on the part of the firm
Sunk Costs and Commitment

- **Sunk costs** are one-time investments that must be made to enter a market
  - these allow the firm to produce in the market but have no residual value if the firm leaves the market
  - could include expenditures on unique types of equipment or job-specific training of workers
First-Mover Advantage in Cournot’s Natural Springs

• Under the Stackelberg version of this model, each firm has two possible strategies
  – be a leader \( (q_i = 60) \)
  – be a follower \( (q_i = 30) \)
First-Mover Advantage in Cournot’s Natural Springs

- The payoffs for these two strategies are:

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<td>(q_A = 60)</td>
<td>Leader (q_B = 60)</td>
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<td>A: 0</td>
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<td></td>
<td>B: 0</td>
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<td></td>
<td>A: $900</td>
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<td></td>
<td>B: $1,800</td>
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<tr>
<td>(q_A = 30)</td>
<td>Follower (q_B = 30)</td>
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<tr>
<td></td>
<td>A: $1,800</td>
</tr>
<tr>
<td></td>
<td>B: $1,600</td>
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</tbody>
</table>
First-Mover Advantage in Cournot’s Natural Springs

• The leader-leader strategy for each firm proves to be disastrous
  – it is not a Nash equilibrium
    • if firm $A$ knows that firm $B$ will adopt a leader strategy, its best move is to be a follower

• A follower-follower choice is profitable for both firms
  – this choice is unstable because it gives each firm an incentive to cheat
First-Mover Advantage in Cournot’s Natural Springs

• With simultaneous moves, either of the leader-follower pairs represents a Nash equilibrium

• But if one firm has the opportunity to move first, it can dictate which of the two equilibria is chosen
  – this is the first-mover advantage
Entry Deterrence (skipped)

• In some cases, first-mover advantages may be large enough to deter all entry by rivals
  – however, it may not always be in the firm’s best interest to create that large a capacity
Entry Deterrence

• With economies of scale, the possibility for profitable entry deterrence is increased

  – if the first mover can adopt a large-enough scale of operation, it may be able to limit the scale of a potential entrant

  • the potential entrant will experience such high average costs that there would be no advantage to entering the market
Entry Deterrence in Cournot’s Natural Spring

- Assume that each spring owner must pay a fixed cost of operations ($784)
- The Nash equilibrium leader-follower strategies remain profitable for both firms
  - if firm A moves first and adopts the leader’s role, B’s profits are relatively small ($116)
    - A could push B out of the market by being a bit more aggressive
Entry Deterrence in Cournot’s Natural Spring

• Since B’s reaction function is unaffected by the fixed costs, firm A knows that
  \[ q_B = \frac{(120 - q_A)}{2} \]
  and market price is given by
  \[ P = 120 - q_A - q_B \]

• Firm A knows that B’s profits are
  \[ \pi_B = Pq_B - 784 \]
Entry Deterrence in Cournot’s Natural Spring

• When $B$ is a follower, its profits depend only on $q_A$.

• Therefore,

$$\pi_B = \left( \frac{120 - q_A}{2} \right)^2 - 784$$

• Firm $A$ can ensure nonpositive profits for firm $B$ by choosing $q_A \geq 64$
  
  – Firm $A$ will earn profits of $2,800$
Limit Pricing (skipped)

• Are there situations where a monopoly might purposely choose a low ("limit") price policy to deter entry into its market?

• In most simple situations, the limit pricing strategy does not yield maximum profits and is not sustainable over time
  - choosing \( P_L < P_M \) will only deter entry if \( P_L \) is lower than the AC of any potential entrant
Limit Pricing

• If the monopoly and the potential entrant have the same costs, the only limit price sustainable is $P_L = AC$
  – defeats the purpose of being a monopoly because $\pi = 0$

• Thus, the basic monopoly model offers little room for entry deterrence through pricing behavior
Limit Pricing and Incomplete Information

• Believable models of limit pricing must depart from traditional assumptions
• The most important set of such models involves incomplete information
  – if an incumbent monopolist knows more about the market situation than a potential entrant, the monopolist may be able to deter entry
Limit Pricing and Incomplete Information

- Suppose that an incumbent monopolist may have either “high” or “low” production costs as a result of past decisions
- The profitability of firm B’s entry into the market depends on A’s costs
- We can use a tree diagram to show B’s dilemma
Limit Pricing and Incomplete Information

The profitability of entry for Firm $B$ depends on Firm $A$’s costs which are unknown to $B$.

The diagram shows the possible outcomes for Firm $A$ and Firm $B$.

- Firm $A$ has two cost scenarios: high and low.
- Firm $B$ can choose to enter or not enter.

The payoffs for each scenario are as follows:

- **High Cost:**
  - If Firm $A$ enters, the payoff for Firm $B$ is 1, the payoff for Firm $A$ is 3.
  - If Firm $B$ does not enter, the payoff for both firms is 4.

- **Low Cost:**
  - If Firm $A$ enters, the payoff for Firm $B$ is 3, the payoff for Firm $A$ is -1.
  - If Firm $B$ does not enter, the payoff for both firms is 6.

These payoffs represent the net profits or utilities for each firm in each scenario.
Limit Pricing and Incomplete Information

• Firm $B$ could use whatever information it has to develop a subjective probability of $A$’s cost structure

• If $B$ assumes that there is a probability of $\rho$ that $A$ has high cost and $(1-\rho)$ that it has low cost, entry will yield positive expected profits if

\[
E(\pi_B) = \rho(3) + (1-\rho)(-1) > 0
\]

\[
\rho > \frac{1}{4}
\]
Limit Pricing and Incomplete Information

• Regardless of its true costs, firm A is better off if B does not enter.
• One way to ensure this is for A to convince B that $\rho < \frac{1}{4}$.
• Firm A may choose a low-price strategy then to signal firm B that its costs are low.
  – this provides a possible rationale for limit pricing.
Predatory Pricing

• The structure of many models of predatory behavior is similar to that used in limit pricing models
  – stress incomplete information
• A firm wishes to encourage its rival to exit the market
  – it takes actions to affect its rival’s views of the future profitability of remaining in the market
Games of Incomplete Information

• Each player in a game may be one of a number of possible types \((t_A \text{ and } t_B)\)
  – player types can vary along several dimensions

• We will assume that our player types have differing potential payoff functions
  – each player knows his own payoff but does not know his opponent’s payoff with certainty
Games of Incomplete Information

• Each player’s conjectures about the opponent’s player type are represented by belief functions \([f_A(t_B)]\)
  – consist of the player’s probability estimates of the likelihood that his opponent is of various types

• Games of incomplete information are sometimes referred to as Bayesian games
Games of Incomplete Information

• We can now generalize the notation for the game

\[G[S_A, S_B, t_A, t_B, f_A, f_B, U_A(a, b, t_A, t_B), U_B(a, b, t_A, t_B)]\]

• The payoffs to A and B depend on the strategies chosen \((a \subset S_A, b \subset S_B)\) and the player types
Games of Incomplete Information

• For one-period games, it is fairly easy to generalize the Nash equilibrium concept to reflect incomplete information
  – we must use expected utility because each player’s payoffs depend on the unknown player type of the opponent
Games of Incomplete Information

- A strategy pair \((a^*, b^*)\) will be a Bayesian-Nash equilibrium if \(a^*\) maximizes \(A\)'s expected utility when \(B\) plays \(b^*\) and vice versa.

\[
E[U_A(a^*, b^*, t_A, t_B)] = \sum_{t_B} f_A(t_B)U(a^*, b^*, t_A, t_B)
\]

\[
\geq E[U_B(a', b^*, t_A, t_B)] \text{ for all } a' \subset S_A
\]

\[
E[U_A(a^*, b^*, t_A, t_B)] = \sum_{t_A} f_B(t_A)U(a^*, b^*, t_A, t_B)
\]

\[
\geq E[U_B(a^*, b', t_A, t_B)] \text{ for all } b' \subset S_B
\]
A Bayesian-Cournot Equilibrium

• Suppose duopolists compete in a market for which demand is given by

\[ P = 100 - q_A - q_B \]

• Suppose that \( MC_A = MC_B = 10 \)
  – the Nash (Cournot) equilibrium is \( q_A = q_B = 30 \) and payoffs are \( \pi_A = \pi_B = 900 \)
A Bayesian-Cournot Equilibrium

• Suppose that $MC_A = 10$, but $MC_B$ may be either high (= 16) or low (= 4)
• Suppose that $A$ assigns equal probabilities to these two “types” for $B$ so that the expected $MC_B = 10$
• $B$ does not have to consider expectations because it knows there is only a single $A$ type
A Bayesian-Cournot Equilibrium

- $B$ chooses $q_B$ to maximize
  \[ \pi_B = (P - MC_B)(q_B) = (100 - MC_B - q_A - q_B)(q_B) \]
- The first-order condition for a maximum is
  \[ q_B^* = (100 - MC_B - q_A)/2 \]
- Depending on $MC_B$, this is either
  \[ q_B^* = (84 - q_A)/2 \quad \text{or} \quad q_B^* = (96 - q_A)/2 \]
A Bayesian-Cournot Equilibrium

• Firm A must take into account that B could face either high or low marginal costs so its expected profit is

\[ \pi_A = 0.5(100 - MC_A - q_A - q_{BH})(q_A) + 0.5(100 - MC_A - q_A - q_{BL})(q_A) \]

\[ \pi_A = (90 - q_A - 0.5q_{BH} - 0.5q_{BL})(q_A) \]
A Bayesian-Cournot Equilibrium

• The first-order condition for a maximum is

\[ q_A^* = (90 - 0.5q_{BH} - 0.5q_{BL})/2 \]

• The Bayesian-Nash equilibrium is:

\[ q_A^* = 30 \]
\[ q_{BH}^* = 27 \]
\[ q_{BL}^* = 33 \]

• These choices represent an \textit{ex ante} equilibrium
Mechanism Design and Auctions (skipped)

• The concept of Bayesian-Nash equilibrium has been used to study auctions
  – by examining equilibrium solutions under various possible auction rules, game theorists have devised procedures that obtain desirable results
    • achieving high prices for the goods being sold
    • ensuring the goods end up with those who value them most
An Oil Tract Auction

• Suppose two firms are bidding on a tract of land that may have oil underground
• Each firm has decided on a potential value for the tract ($V_A$ and $V_B$)
• The seller would like to obtain the largest price possible for the land
  – the larger of $V_A$ or $V_B$
• Will a simple sealed bid auction work?
An Oil Tract Auction

• To model this as a Bayesian game, we need to model each firm’s beliefs about the other’s valuations
  – $0 \leq V_i \leq 1$
  – each firm assumes that all possible values for the other firm’s valuation are equally likely
    • firm A believes that $V_B$ is uniformly distributed over the interval $[0,1]$ and vice versa
An Oil Tract Auction

• Each firm must now decide its bid ($b_A$ and $b_B$)

• The gain from the auction for firm $A$ is

$$V_A - b_A \text{ if } b_A > b_B$$

and

$$0 \text{ if } b_A < b_B$$

• Assume that each player opts to bid a fraction ($k_i$) of the valuation
An Oil Tract Auction

- Firm A’s expected gain from the sale is
  \[ \pi_A = (V_A - b_A) \cdot \text{Prob}(b_A > b_B) \]
- Since A believes that \( V_B \) is distributed normally,
  \[
  \text{prob}(b_A > b_B) = \text{prob}(b_A > k_B V_B) \\
  = \text{prob}(b_A / k_B > V_B) = b_A / k_B
  \]
- Therefore,
  \[ \pi_A = (V_A - b_A) \cdot (b_A / k_B) \]
An Oil Tract Auction

• Note that $\pi_A$ is maximized when
  \[ b_A = \frac{V_A}{2} \]

• Similarly,
  \[ b_B = \frac{V_B}{2} \]

• The firm with the highest valuation will win the bid and pay a price that is only 50 percent of the valuation.
An Oil Tract Auction

• The presence of additional bidders improves the situation for the seller

• If firm $A$ continues to believe that each of its rivals’ valuations are uniformly distributed over the $[0,1]$ interval,

$$\text{prob}(b_A > b_i) = \text{prob}(b_A > k_i V_i) \text{ for } i = 1, \ldots, n$$

$$= \prod_{i=1}^{n-1} \left( \frac{b_A}{k_i} \right) = b_A^{n-1} / k^{n-1}$$
An Oil Tract Auction

• This means that

\[ \pi_A = (V_A - b_A)(b_A^{n-1}/k^{n-1}) \]

and the first-order condition for a maximum is

\[ b_A = [(n-1)/n]V_A \]

• As the number of bidders rises, there are increasing incentives for a truthful revelation of each firm’s valuation.
Dynamic Games with Incomplete Information

• In multiperiod and repeated games, it is necessary for players to update beliefs by incorporating new information provided by each round of play.

• Each player is aware that his opponent will be doing such updating—must take this into account when deciding on a strategy.
Important Points to Note:

• All games are characterized by similar structures involving players, strategies available, and payoffs obtained through their play
  – the Nash equilibrium concept provides an attractive solution to a game
    • each player’s strategy choice is optimal given the choices made by the other players
    • not all games have unique Nash equilibria
Important Points to Note:

• Two-person noncooperative games with continuous strategy sets will usually possess Nash equilibria
  – games with finite strategy sets will also have Nash equilibria in mixed strategies
Important Points to Note:

• In repeated games, Nash equilibria that involve only credible threats are called subgame-perfect equilibria.
Important Points to Note:

• In a simple single-period game, the Nash-Bertrand equilibrium implies competitive pricing with price equal to marginal cost.

• The Cournot equilibrium (with $p > mc$) can be interpreted as a two-stage game in which firms first select a capacity constraint.
Important Points to Note:

• Tacit collusion is a possible subgame-perfect equilibrium in an infinitely repeated game

  – the likelihood of such equilibrium collusion diminishes with larger numbers of firms, because the incentive to chisel on price increases
Important Points to Note:

• Some games offer first-mover advantages
  – in cases involving increasing returns to scale, such advantages may result in the deterrence of all entry
Important Points to Note:

• Games of incomplete information arise when players do not know their opponents’ payoff functions and must make some conjectures about them
  – in such Bayesian games, equilibrium concepts involve straightforward generalizations of the Nash and subgame-perfect notions encountered in games of complete information